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Effective gauge theory in Spintronics

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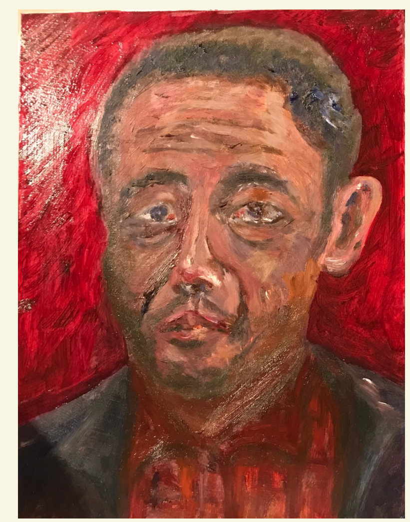

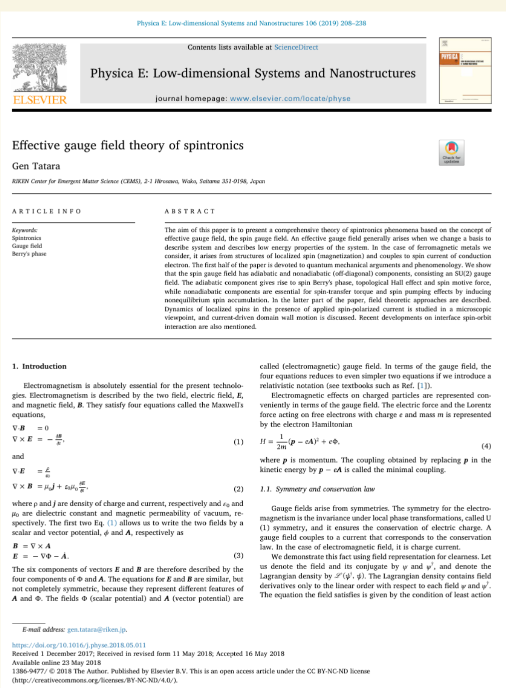

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Effective gauge field theory of spintronics

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ABSTRACT

The aim of this paper is to present a comprehensive theory of spintronics phenomena based on the concept of effective gauge field, the spin gauge field. An effective gauge field generally arises when we change a basis to describe system and describes low energy properties of the system. In the case of ferromagnetic metals we consider, it arises from structures of localized spin (magnetization) and couples to spin current of conduction electron. The first half of the paper is devoted to quantum mechanical arguments and phenomenology. We show that the spin gauge field has adiabatic and nonadiabatic (off-diagonal) components, consisting an SU(2) gauge field. The adiabatic component gives rise to spin Berry's phase, topological Hall effect and spin motive force, while nonadiabatic components are essential for spin-transfer torque and spin pumping effects by inducing nonequilibrium spin accumulation. In the latter part of the paper, field theoretic approaches are described. Dynamics of localized spins in the presence of applied spin-polarized current is studied in a microscopic viewpoint, and current-driven domain wall motion is discussed. Recent developments on interface spin-orbit interaction are also mentioned.

1. Introduction

Electromagnetism is absolutely essential for the present technologies. Electromagnetism is described by the two field, electric field, $\boldsymbol{E}$, and magnetic field, $\boldsymbol{B}$. They satisfy four equations called the Maxwell's equations,

$$\begin{aligned} \nabla\cdot\boldsymbol{B} &= 0 \\ \nabla\times\boldsymbol{E} &= -\frac{\partial B}{\partial t}, \end{aligned} \tag{1}$$

and

$$\begin{aligned} \nabla\cdot\boldsymbol{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla\times\boldsymbol{B} &= \mu_0\boldsymbol{j} + \varepsilon_0\mu_0\frac{\partial E}{\partial t}, \end{aligned} \tag{2}$$

where ρ and $\boldsymbol{j}$ are density of charge and current, respectively and ε_0 and μ_0 are dielectric constant and magnetic permeability of vacuum, respectively. The first two Eq. (1) allows us to write the two fields by a scalar and vector potential, ϕ and $\boldsymbol{A}$, respectively as

$$\begin{aligned} \boldsymbol{B} &= \nabla\times\boldsymbol{A} \\ \boldsymbol{E} &= -\nabla\Phi - \dot{\boldsymbol{A}}. \end{aligned} \tag{3}$$

The six components of vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ are therefore described by the four components of Φ and $\boldsymbol{A}$. The equations for $\boldsymbol{E}$ and $\boldsymbol{B}$ are similar, but not completely symmetric, because they represent different features of $\boldsymbol{A}$ and Φ. The fields Φ (scalar potential) and $\boldsymbol{A}$ (vector potential) are called (electromagnetic) gauge field. In terms of the gauge field, the four equations reduces to even simpler two equations if we introduce a relativistic notation (see textbooks such as Ref. [1]).

Electromagnetic effects on charged particles are represented conveniently in terms of the gauge field. The electric force and the Lorentz force acting on free electrons with charge e and mass m is represented by the electron Hamiltonian

$$H = \frac{1}{2m}(\boldsymbol{p} - e\boldsymbol{A})^2 + e\Phi, \tag{4}$$

where $\boldsymbol{p}$ is momentum. The coupling obtained by replacing $\boldsymbol{p}$ in the kinetic energy by $\boldsymbol{p} - e\boldsymbol{A}$ is called the minimal coupling.

1.1. Symmetry and conservation law

Gauge fields arise from symmetries. The symmetry for the electromagnetism is the invariance under local phase transformations, called U(1) symmetry, and it ensures the conservation of electric charge. A gauge field couples to a current that corresponds to the conservation law. In the case of electromagnetic field, it is charge current.

We demonstrate this fact using field representation for clearness. Let us denote the field and its conjugate by ψ and $\psi^\dagger$, and denote the Lagrangian density by $\mathscr{L}(\psi^\dagger, \psi)$. The Lagrangian density contains field derivatives only to the linear order with respect to each field ψ and $\psi^\dagger$. The equation the field satisfies is given by the condition of least action

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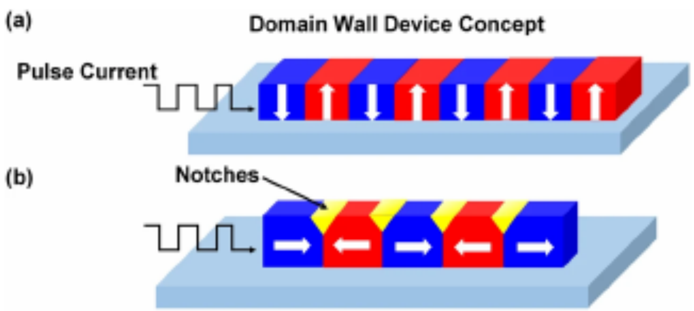
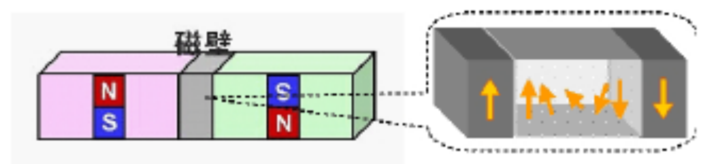
Reference G. TATARA, Physica E: low dim. $\underline{106}$, 208 (2019)

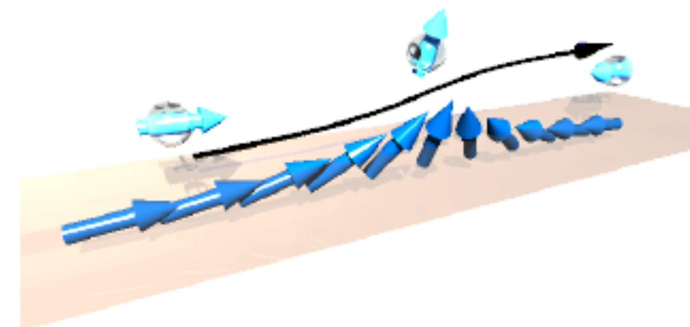
Gauge field in spintronics

Non-trivial effect emerges from magnetization spin structures

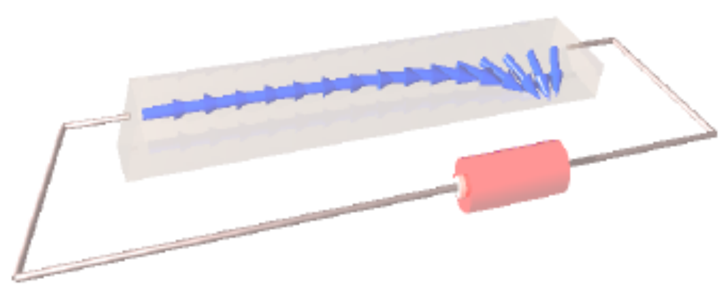
- Domain wall



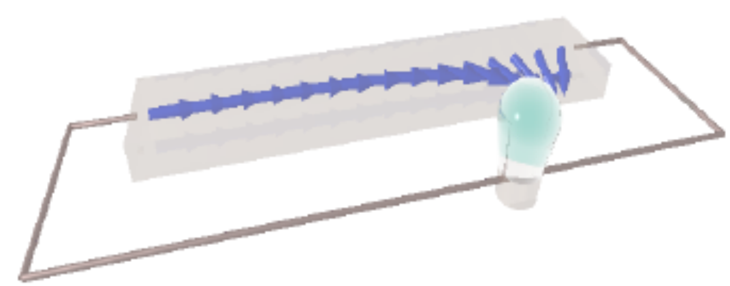




- DW 'pushed' by electron
- Electron pushed by DW

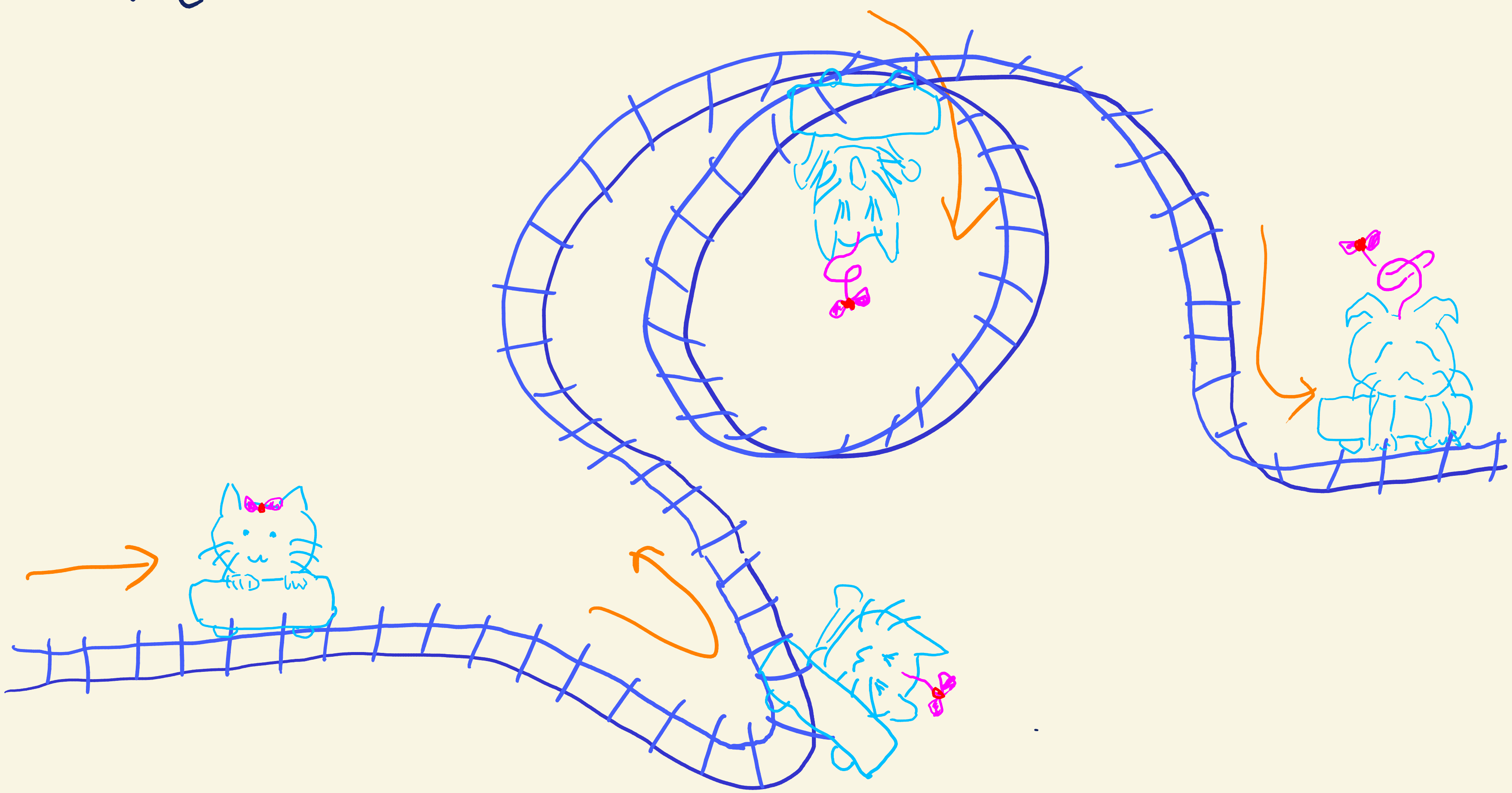


Write information by current



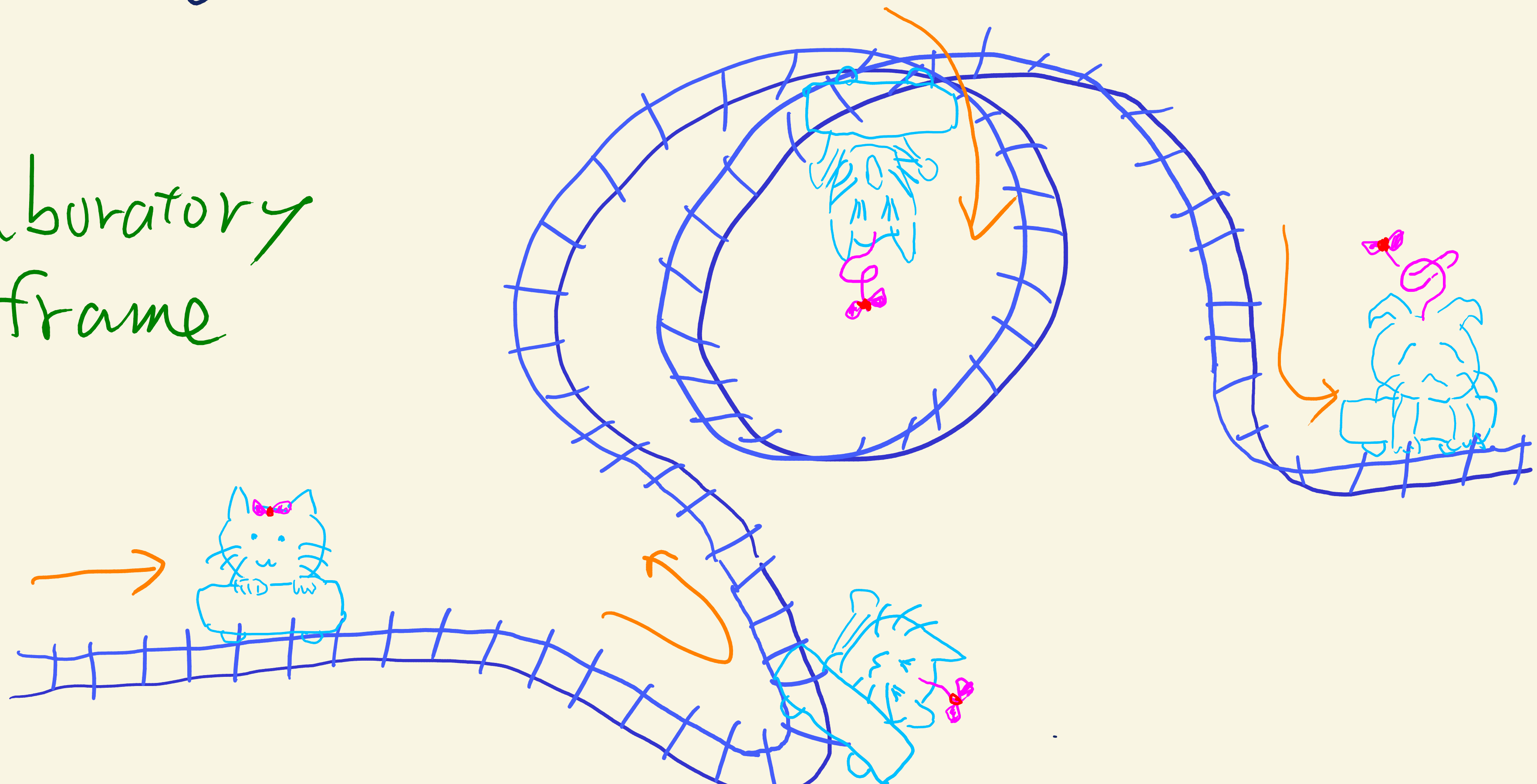
Read information, Spin battery

What we learn

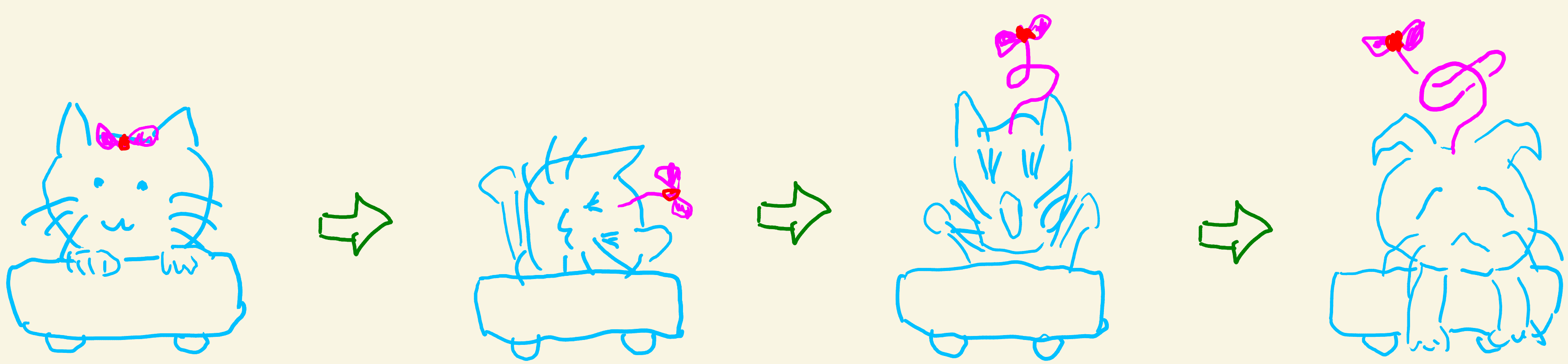


What we learn

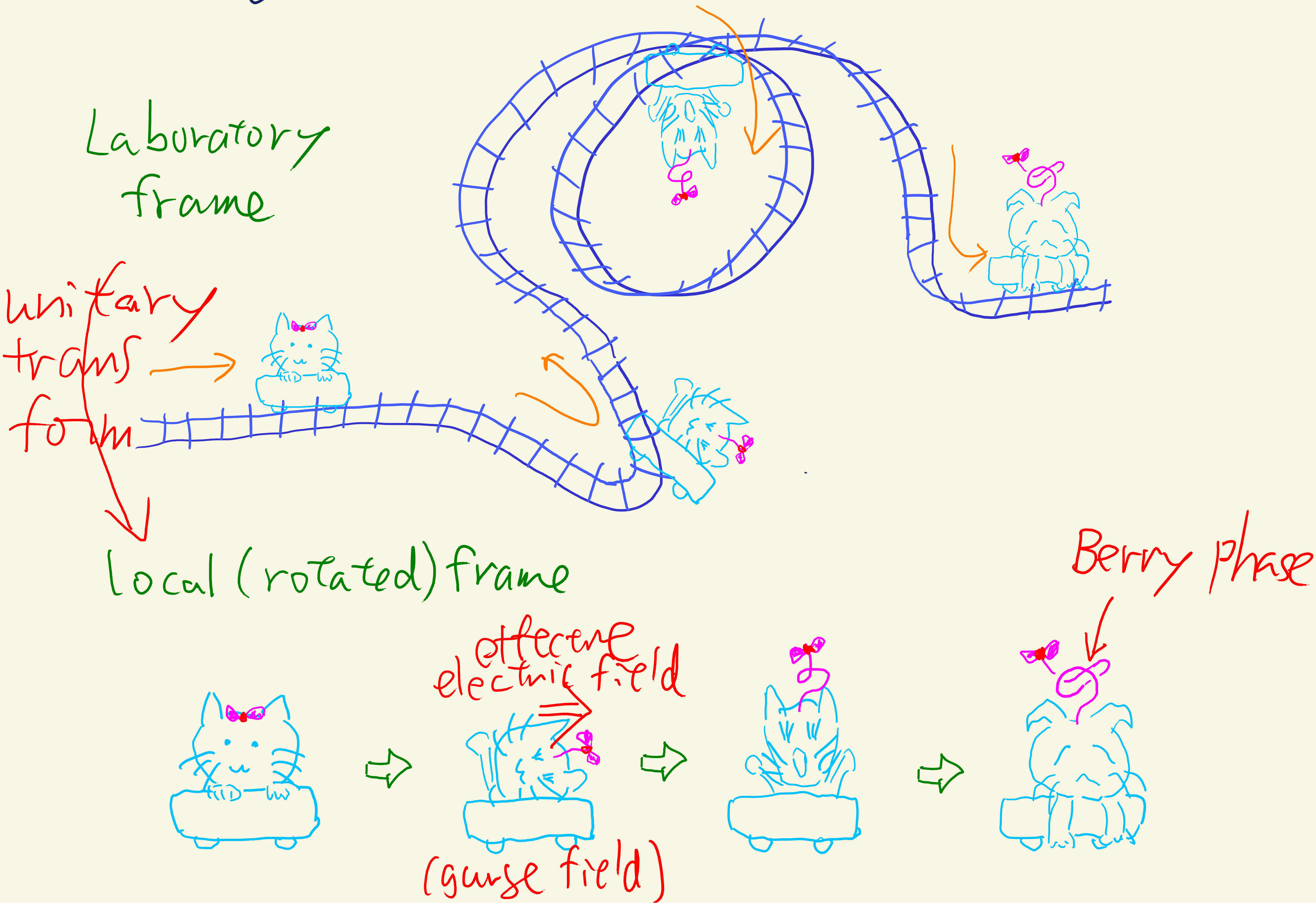
Laboratory frame



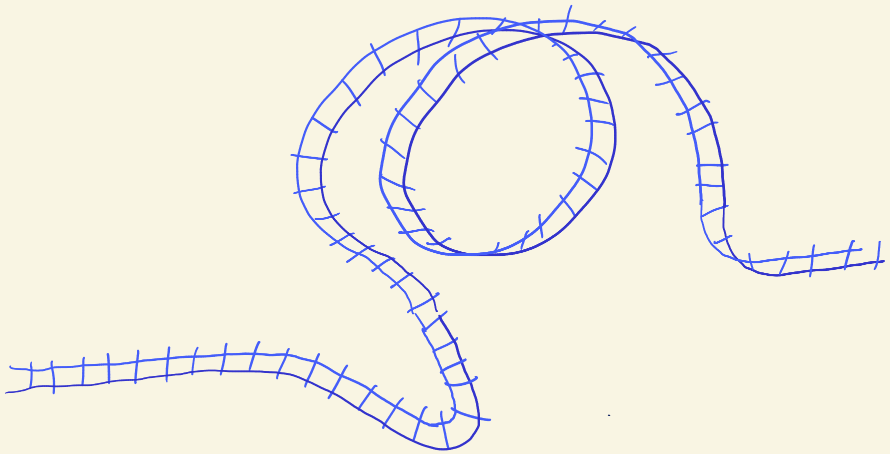
local (rotated) frame



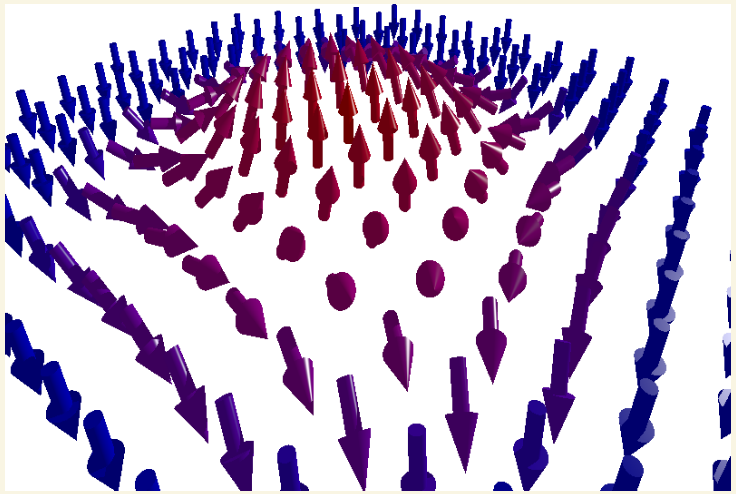
What we learn



Spintronics : manipulation of $\begin{cases} \text{magnetism electrically} \\ \text{electronics magnetically} \end{cases}$



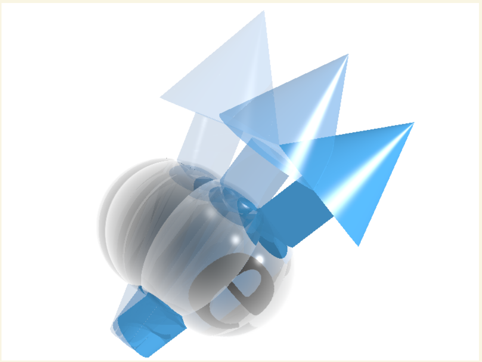
$\Rightarrow$



$\mathbb{S}(\mathbf{r}, t)$

local spin structure
(magnetization)

$\Rightarrow$



σ

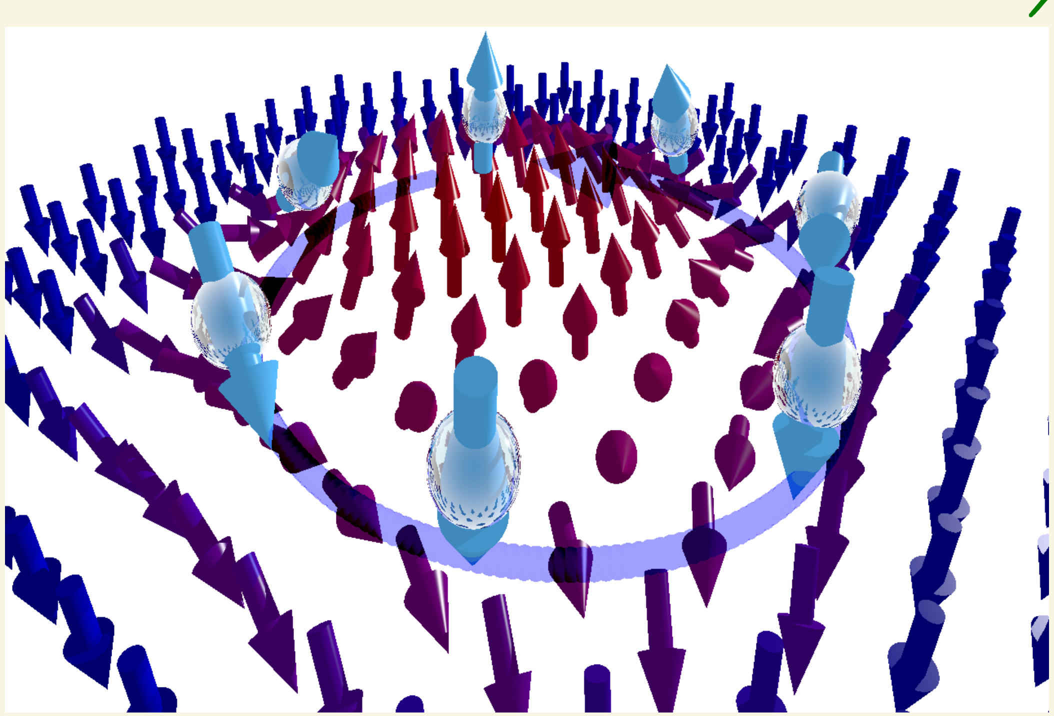
conduction electron
with spin $\frac{1}{2}$

conduction electron in spin structure

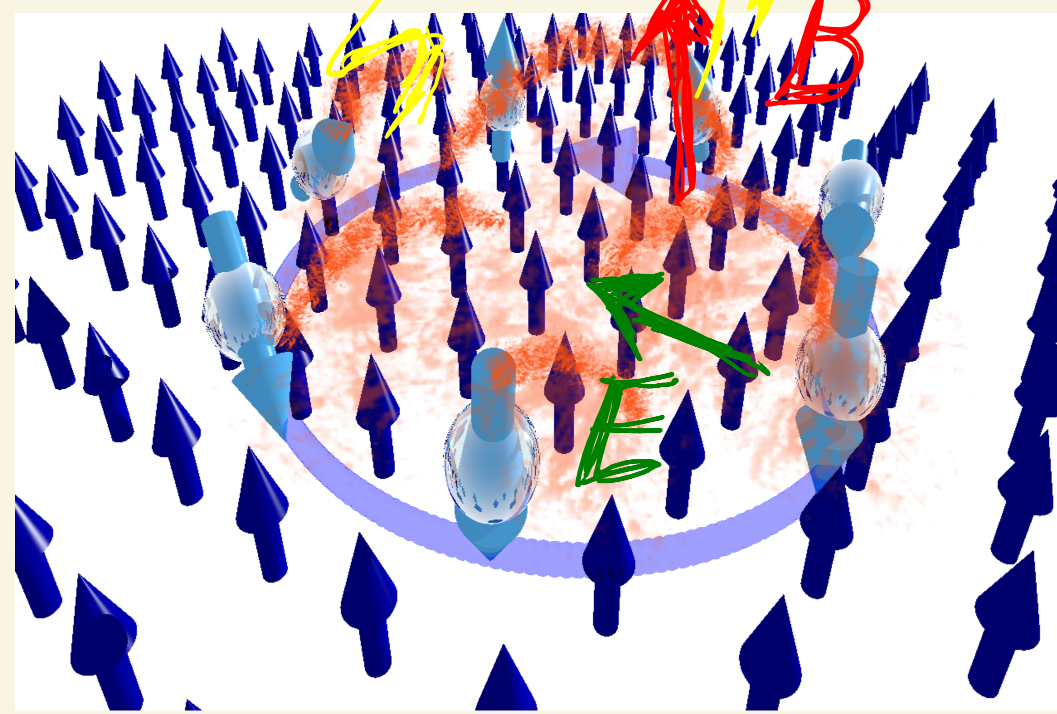
effective gauge field (electromagnetism)

unitary transformation

photon



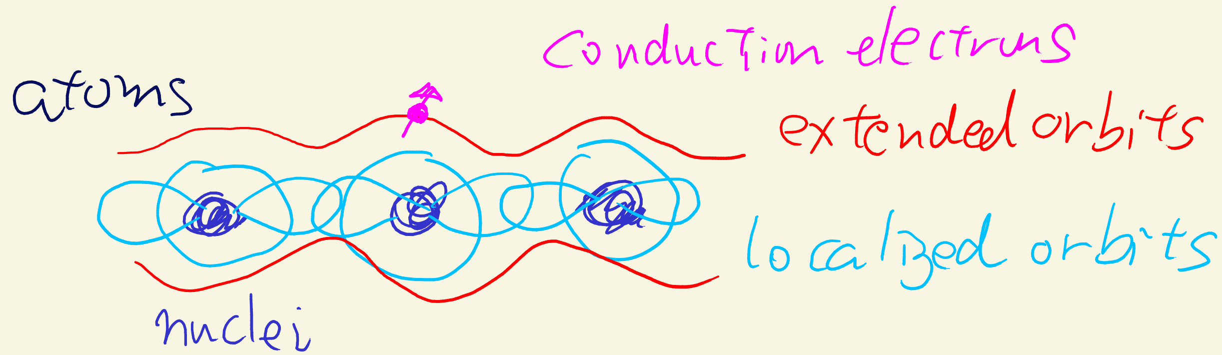
⇒



Metal

Conducting $\Rightarrow$ conduction electron $\simeq$ free electron

• non-relativistic



Quantum mechanical Hamiltonian

$$H = -\frac{\hbar^2\nabla^2}{2m} + V(r) \simeq -\frac{\hbar^2\nabla^2}{2m^*}$$

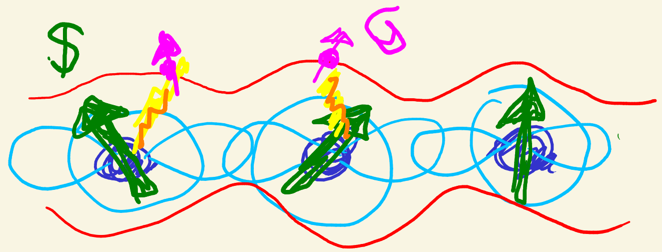
$m^* \simeq m$

lattice potential
periodic

free with effective mass m^*

Magnets

· localized spin $\mathbf{S}(r)$ couples to electron spin $\boldsymbol{\sigma}$



sd exchange coupling

scalar coupling

$$H_{sd} = J\,\mathbf{S}\cdot\boldsymbol{\sigma}$$

Ferromagnet $\mathbf{S}(r) \sim S\hat{z}$
uniform

Antiferromagnet $\mathbf{S}(r) \sim S(-)^{r}$

Quantum mechanics in metallic magnets
(Electron)

$$H = \frac{-\hbar^2\nabla^2}{2m} + J\,\mathbf{S}(r)\cdot\boldsymbol{\sigma}$$

Field theory

$$\Rightarrow H_{FT} = \int d^3r\; c^\dagger(r)\left[\frac{-\hbar^2\nabla^2}{2m} + J\,\mathbf{S}\cdot\boldsymbol{\sigma}\right]c(r)$$

$n = c^\dagger c(r)$: electron density

$$|\psi\rangle = \begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix} \quad \text{2 component state}$$

$\{c(r), c^\dagger(r')\} \equiv cc^\dagger + c^\dagger c = \delta(r-r')$

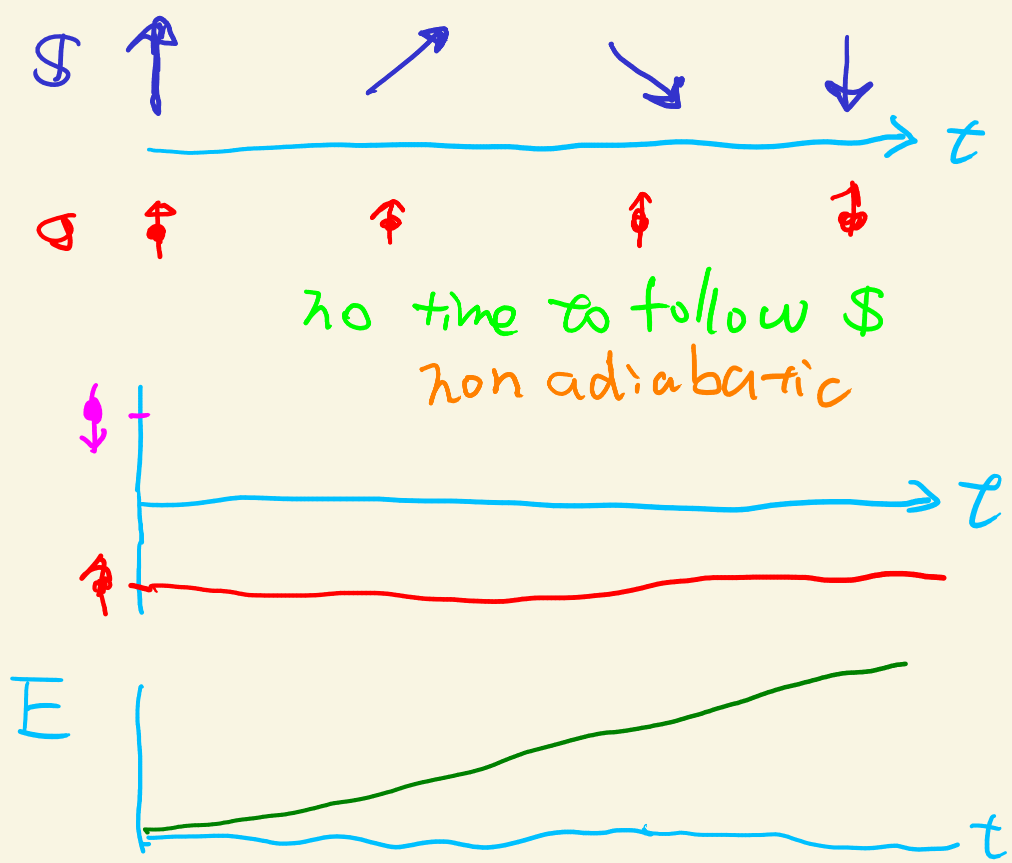
anti commutation relation
(Fermion)

Single localized spin

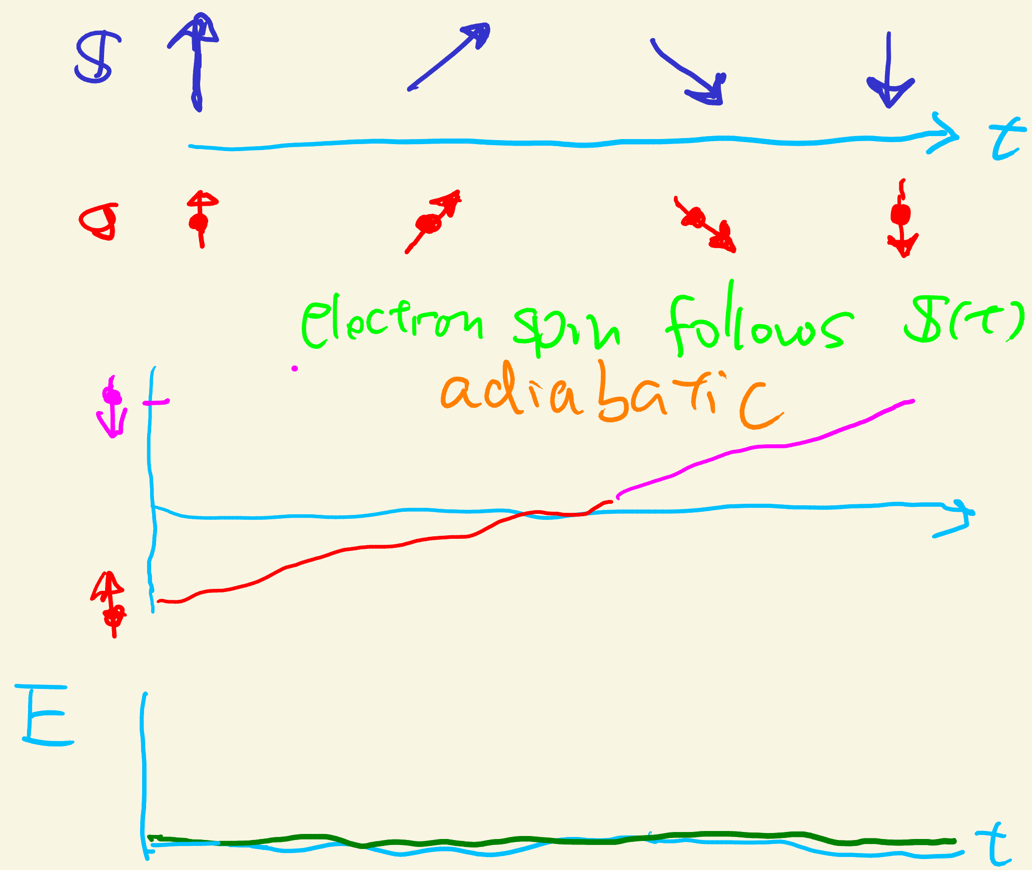
$$H = J\, \mathbf{S}(t)\cdot\boldsymbol{\sigma}$$

$J<0$ time-dependent

- Fast change of $S(t)$



- Slow



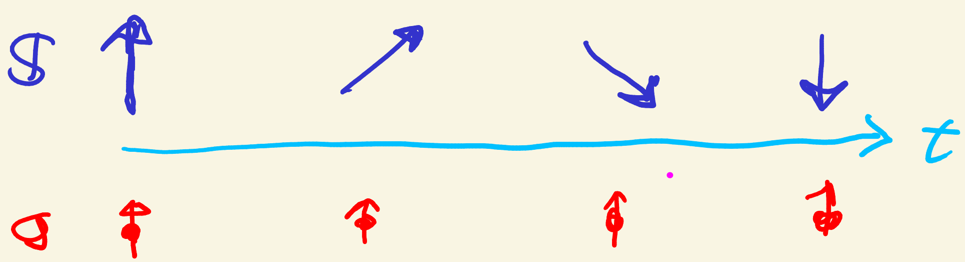
This is not all !

Single localized spin

$$H = J\, \mathbf{S}(t) \cdot \boldsymbol{\sigma}$$

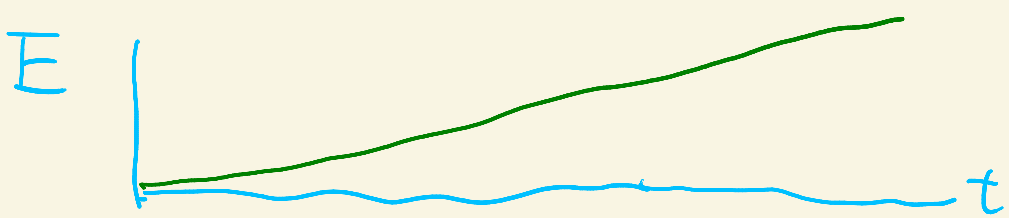
$J<0$ time-dependent

• Fast change of $S(t)$

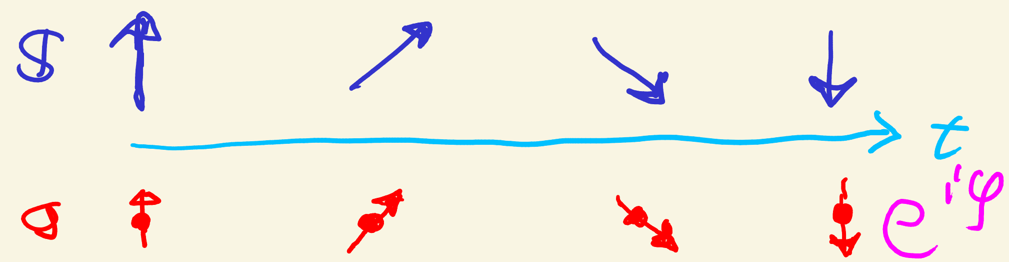


no time to follow S

non adiabatic

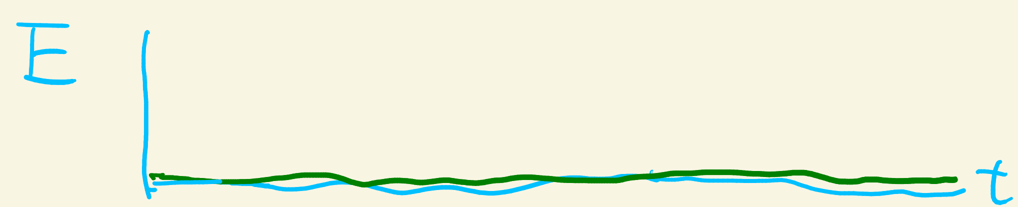


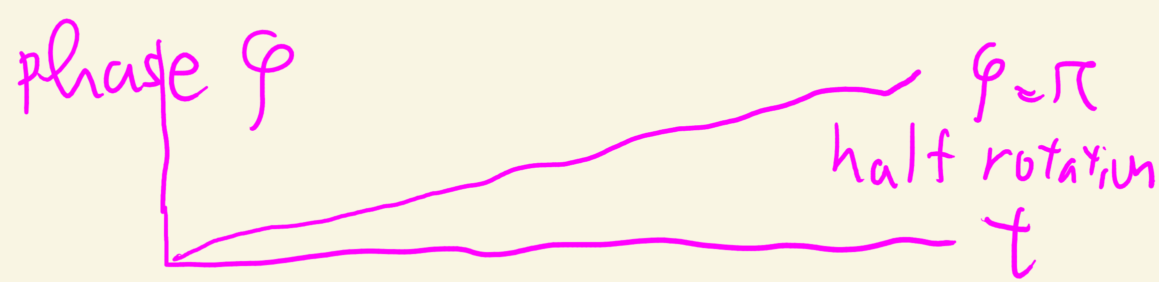
• Slow



electron spin follows $S(t)$

adiabatic





phase due to spin rotation

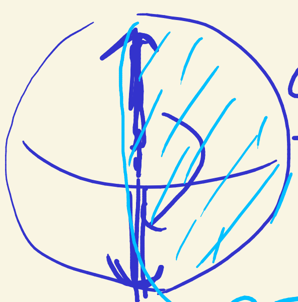
Spin Berry phase

For closed path

$e^{i\varphi}$ depending on path

φ = solid angle $\times S$

· Half rotation



2π

$$\varphi = e^{2\pi S} = \begin{cases} 1 & S = \text{integer } 0, 1, 2 \dots \\ -1 & \text{half integer } \frac{1}{2}, \frac{3}{2} \dots \end{cases}$$

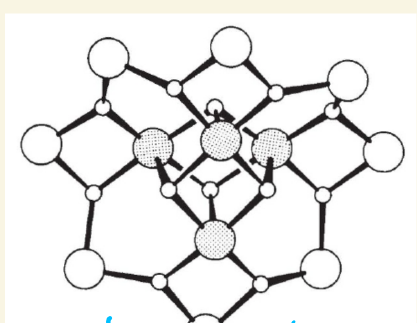
• Haldane gap

1D AF spin chain due to spin Berry phase 1983

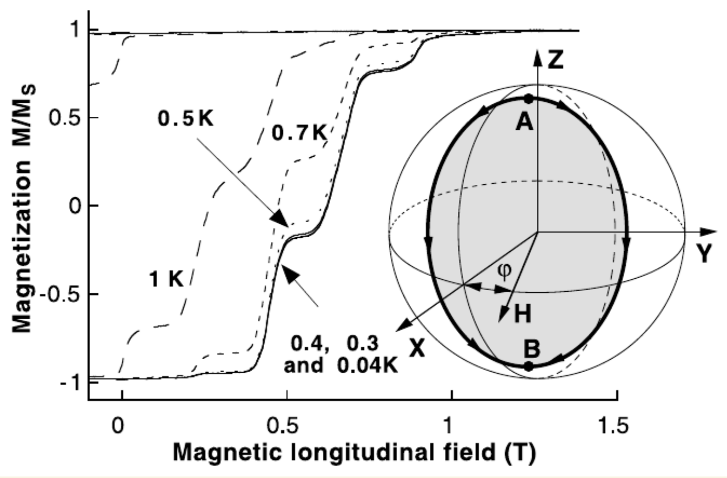
• Molecular magnet

Mn12 $S = 10$

flip rate modulated by spin Berry phase

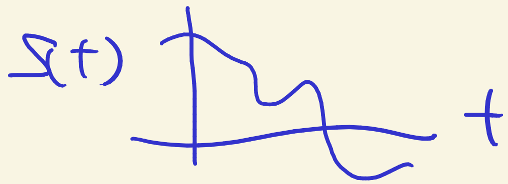


Sessoli 1999



Derivation of Spin Berry phase

- $H = J\,\mathbf{S}(t)\cdot\boldsymbol{\sigma}$ $\quad |\psi(t)\rangle = \begin{pmatrix}\psi_\uparrow(t)\\ \psi_\downarrow(t)\end{pmatrix}$ electron spin wf



generally, $\mathbf{S}\cdot\boldsymbol{\sigma}$ has off-diagonal

- Schrödinger equation

$$i\partial_t|\psi\rangle = J\,\mathbf{S}(t)\cdot\boldsymbol{\sigma}|\psi\rangle$$

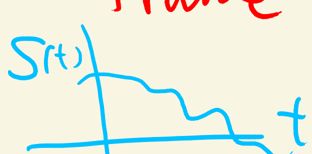
- rotated frame diagonalized at each time

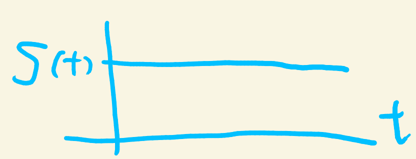
$$U^{-1}(t)\,\mathbf{S}(t)\cdot\boldsymbol{\sigma}\,U(t) = S\sigma_z$$

$U(t)$: 2×2 unitary matrix

$$|\psi(t)\rangle = U(t)|\phi(t)\rangle \quad \Rightarrow \quad i\partial_t U|\phi\rangle = H(t)\,U|\phi\rangle$$

laboratory frame rotated frame





$$i\,U\left(\partial_t + (U^{-1}\partial_t U)\right)|\phi\rangle$$

$$= i\,U[\partial_t + iA_t]|\phi\rangle$$ time component of gauge field

$$A_t = -i\,U^{-1}\partial_t U$$ scalar potential

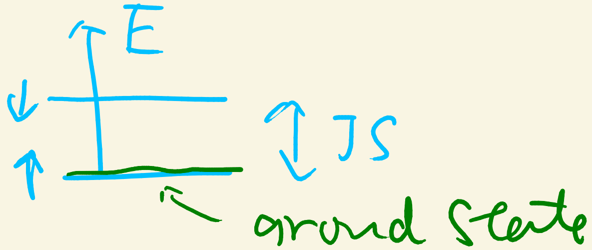
Schrödinger equation in the rotated frame

$$i(\partial_t + iA_t)|\phi\rangle = \tilde{H}|\phi\rangle$$

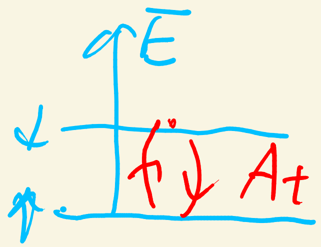
$$\tilde{H} = U^{-1} H U = JS\sigma_z$$ diagonalized

$$A_t = -iU^{-1}\partial_t U$$

- if $\partial_t U \sim 0$



- with A_t



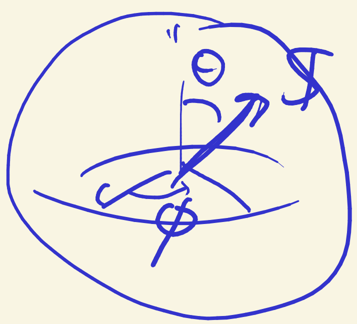
↑ & ↓ states mixed by A_t

time-dependent external field $\mathcal{S}(t)$

- Explicit form of A_t

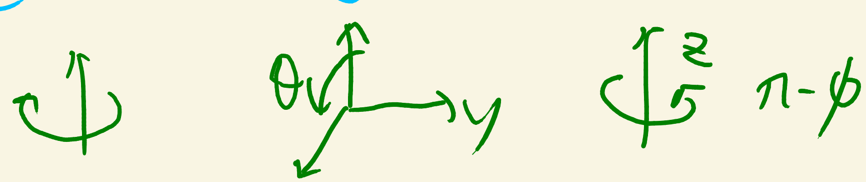
$$U^{-1}(\mathcal{S}\cdot\sigma)U = S\sigma_z \quad \cdots (*)$$

polar coordinate



$$\mathcal{S} = S\begin{pmatrix}\sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta\end{pmatrix}, \quad \mathcal{S}\cdot\sigma = S\begin{pmatrix}\cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta\end{pmatrix}$$

$$U = e^{\frac{\pi}{2}i} e^{-\frac{i}{2}\phi\sigma_z} e^{-\frac{i}{2}\theta\sigma_y} e^{-\frac{i}{2}(\pi-\phi)\sigma_z} = \begin{pmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2}e^{i\phi} & -\cos\frac{\theta}{2}\end{pmatrix}$$



* check that this U satisfies (*)

$$\Rightarrow A_t = -i U^{-1} \partial_t U$$

$$= \frac{1}{2}\Big[(-\partial_t\theta \sin\phi - \sin\theta\cos\phi\,\partial_t\phi)\sigma_x + (\partial_t\theta\cos\phi - \sin\theta\sin\phi\,\partial_t\phi)\sigma_y + (1-\cos\theta)\partial_t\phi\,\sigma_z\Big]$$

$$= \frac{1}{2}\begin{bmatrix} (1-\cos\theta)\partial_t\phi & (-i\partial_t\theta - \sin\theta\,\partial_t\phi)e^{-i\phi} \\ (i\partial_t\theta - \sin\theta\,\partial_t\phi)e^{i\phi} & -(1-\cos\theta)\partial_t\phi \end{bmatrix}$$

· Solution of $i(\partial_t + iA_t)|\phi\rangle = JS\sigma_z|\phi\rangle$

$$|\phi(t=0)\rangle = |\uparrow\rangle \Rightarrow |\phi(t)\rangle = \underbrace{T e^{-i\int_0^t dt' A_t(t')}}_{\text{"phase" of } 2\times 2 \text{ matrix}} |\uparrow\rangle$$

If $JS \gg \hbar\Omega$ high energy state ($\downarrow$) is neglected

frequency of $\dot{S}$

$$\Rightarrow A_t \simeq \langle\uparrow|A_t|\uparrow\rangle = \frac{1}{2}(1-\cos\theta)\dot{\phi}$$

phase

(spin Berry phase)

$\uparrow$

S of electron spin

$$|\phi(t)\rangle = e^{-i\varphi(t)}|\uparrow\rangle$$

$$\varphi(t) = \int_0^t dt' A_t(t')$$

• Electrons in the rotated frame $|\phi\rangle$

feels an effective gauge field (time component)

$$i(\partial_t + iA_t)|\phi\rangle = \tilde{H}|\phi\rangle$$

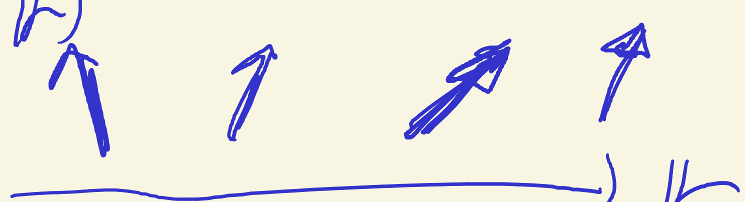
time-dependence $S(t)$

For electron moving around (with kinetic energy)

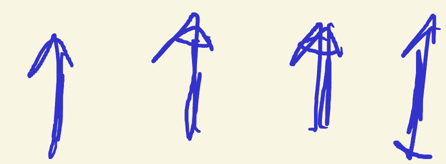
• Spatial component A_i also exists

spatial dependence $S(r)$

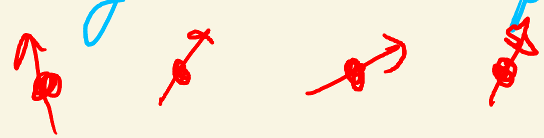
$S(r)$

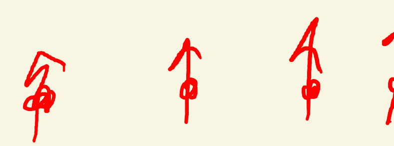


unitary tr. $U(r)$ $\Longrightarrow$



strong sd coupling (adiabatic)





electron spin rotates spatially

spatial component of gauge field

$$\Rightarrow \nabla|\psi\rangle = \nabla U(r)|\phi\rangle = U[\nabla - iA]|\phi\rangle$$

covariant derivative

$$A = iU^{-1}\nabla U$$

Kinetic energy of electron

$$-\frac{\hbar^2}{2m}\nabla^2 \longrightarrow -\frac{\hbar^2}{2m}(\nabla - iA)^2$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V$$

Full Schrödinger equation in rotated frame

$$i\hbar(\partial_t + iA_t)|\phi\rangle = \Big(-\frac{\hbar^2}{2m}(\nabla - iA)^2 + \underbrace{\tilde{V}}_{U^{-1}VU}\Big]|\phi\rangle$$

Effective Hamiltonian

$$\boxed{H = -\frac{\hbar^2}{2m}(\nabla - iA)^2 + \tilde{V} + \hbar A_t}$$

for 2-component electron

$$A_\mu = -iU^{-1}\partial_\mu U \qquad \mu = t, x, y, z$$

SU(2) gauge field (2x2 matrix)

$\downarrow$ adiabatic limit (only $|\uparrow\rangle$ component)

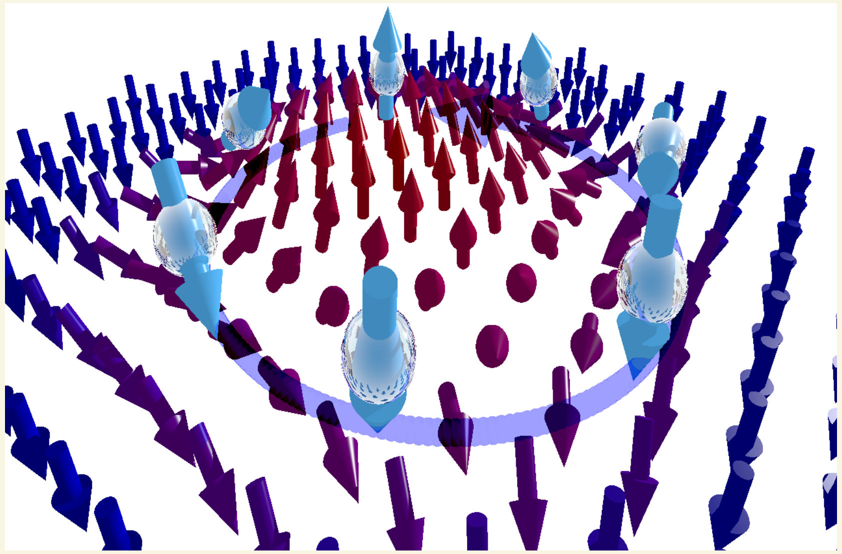
$$\boxed{H = -\frac{\hbar^2}{2m}(\nabla - iA^z)^2 + \tilde{V} + \hbar A_t^z}$$

full gauge field (vector / scalar potential)

$$A_\mu^z = \frac{1}{2}\mathrm{tr}[\sigma_z A_\mu] = \frac{1}{2}(1-\cos\theta)\partial_\mu\phi$$

Effective electromagnetism in ferromagnetic metal

adiabatic limit



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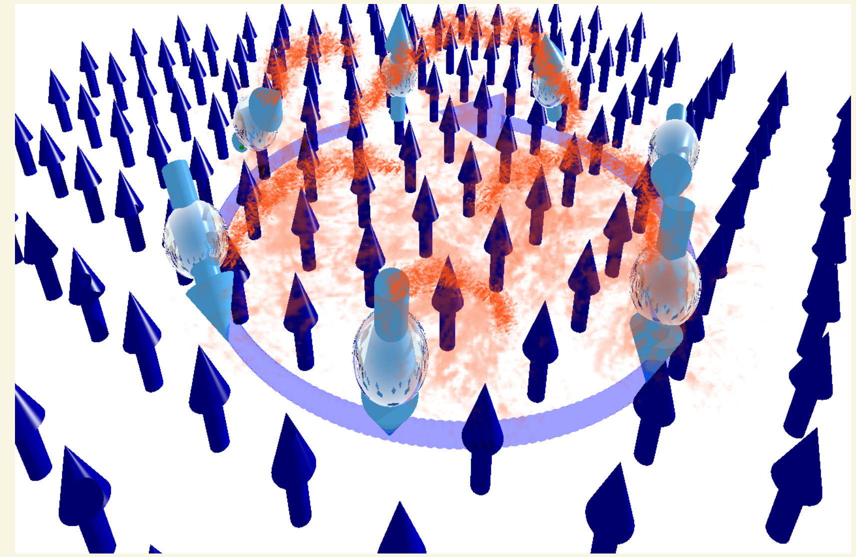


Spin structure = Uniform spin + gauge field

effective U(1) gauge field $\Longrightarrow$ effective electric field, magnetic field

$A_{s,\mu} \, (= A^z_\mu)$

||

different from the electromagnetism

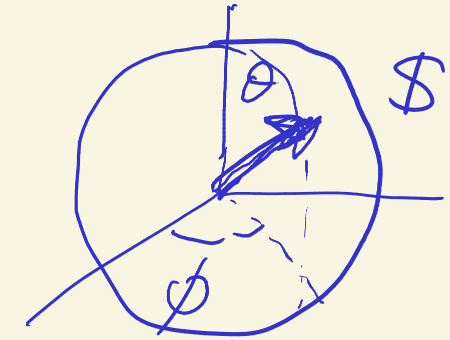
but the same mathematical structure

U(1) gauge invariance

$$E_s = -\nabla A_{s,t} - \partial_t A_s$$

$$B_s = \nabla \times A_s$$

Electric and magnetic fields



$$B_s = \nabla\times A_s , \quad A_s = \frac{1}{2}(1-\cos\theta)\nabla\phi$$

$$B_{s,i} = -\frac{1}{2}\varepsilon_{ijk}\sin\theta\,\nabla_j\theta\,\nabla_k\phi$$

$$= -\frac{1}{4}\varepsilon_{ijk}\, n\cdot(\nabla_j n\times\nabla_k n)$$

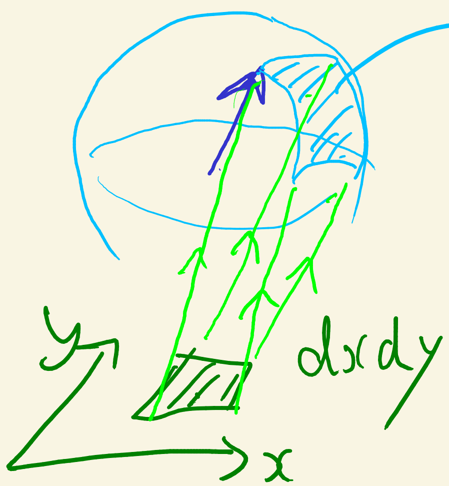
$\sin\theta(\nabla_x\theta\nabla_y\phi - \nabla_y\theta\nabla_x\phi)$

difference of solid angle

2D space

$\nabla_x n\times\nabla_y n$: area of n surface

area $(\nabla_x n\times\nabla_y n)\,dx\,dy$



spin space

real space

B_s is a surface area in spin space

solid angle

$$E_s = \frac{1}{2} n\cdot(\dot{n}\times\nabla n)$$

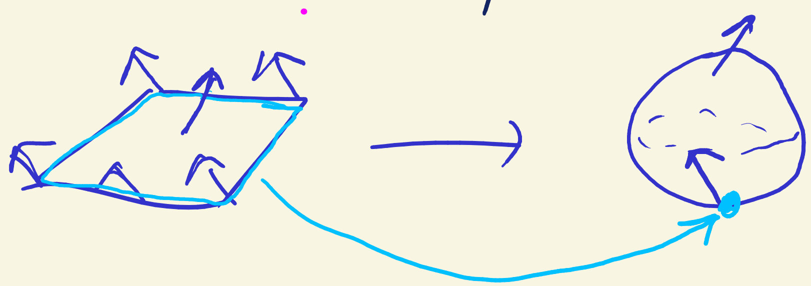
space-time Berry phase

$$\int_S d\mathbf{S}\cdot\mathbf{B}_s = \oint_C d\mathbf{r}\cdot\mathbf{A}_s$$

geometric origin

Solid angle

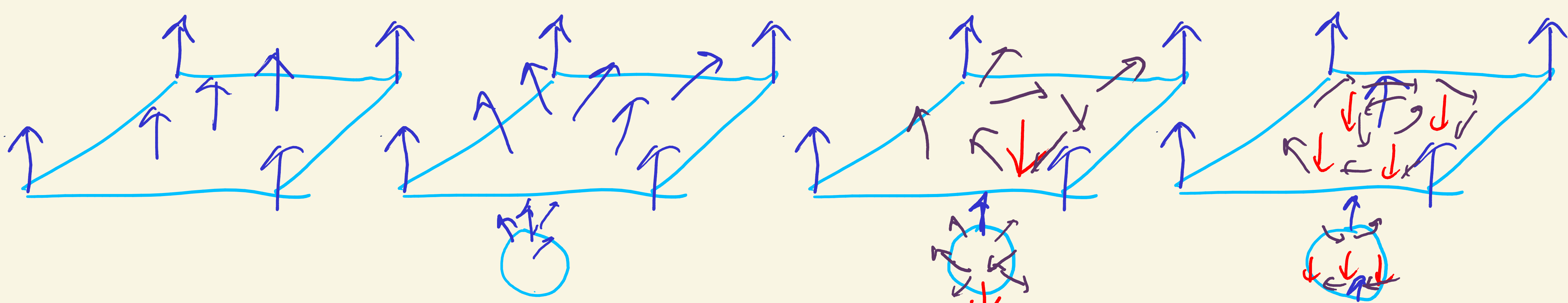
If $\mathbf{S}$ is the same for $|\mathbf{r}| = \infty$,
xy plane is a sphere topologically



$$\Rightarrow \int d\mathbf{S}\cdot\mathbf{B}_S = 4\pi n$$

n: integer
winding number

space spin
$S_2 \to S_2$



$$\int dS \cdot B_s = 4\pi n \Rightarrow$$ monopole in this electromagnetism

$$\int dV (\nabla \cdot B_s)$$

$\Rightarrow \nabla \cdot B_s \neq 0$!

but $\nabla \cdot B_s = \varepsilon_{ijk} \nabla_i (\sin\theta \nabla_j \theta \nabla_k \phi) = 0$!

$\Rightarrow \nabla \cdot B_s \neq 0$ at singularity — locally invisible

e.g. $\theta = \pi$, $\nabla \phi \to \infty$

global object

topological monopole

Maxwell equation of effective electromagnetic field

Coupling to electron spin

$$\boxed{\begin{aligned} \nabla \cdot E_s &= \frac{1}{\varepsilon_s} \rho_s \\ \nabla \times E_s &= -\dot{B} \\ \nabla \cdot B_s &= \rho_m \\ \nabla \times B_s &= \mu_s j_s + \mu_s \varepsilon_s \dot{E}_s \end{aligned}}$$

spin density (ρ_s)

monopole density (ρ_m)

always when U(1) gauge invariance exists

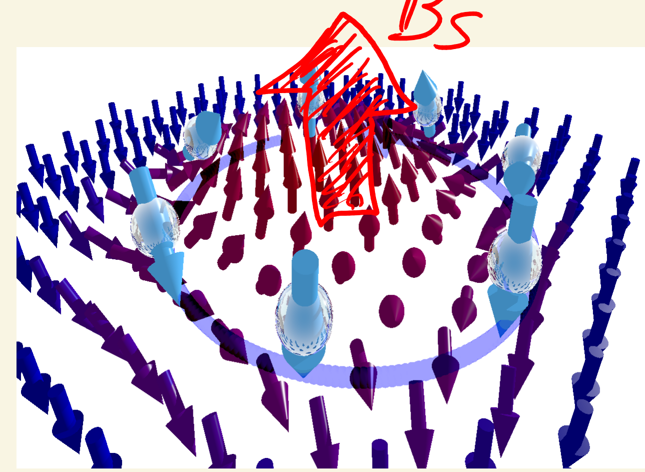
Maxwell equation is derived by transport calculation without knowing electromagnetism !

(spin) charge conservation $\Leftrightarrow$ U(1) gauge theory

Observable effects

$$B_{s_i} = -\frac{1}{4}\varepsilon_{ijk}\, n\cdot(\nabla_j n \times \nabla_k n)$$

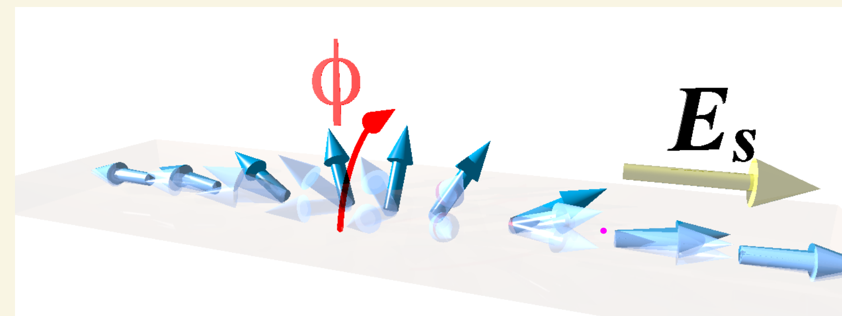
$\Rightarrow$ (spin) Hall effect



$$E_{s_i} = \frac{1}{2}\, n\cdot(\dot{n} \times \nabla_i n)$$

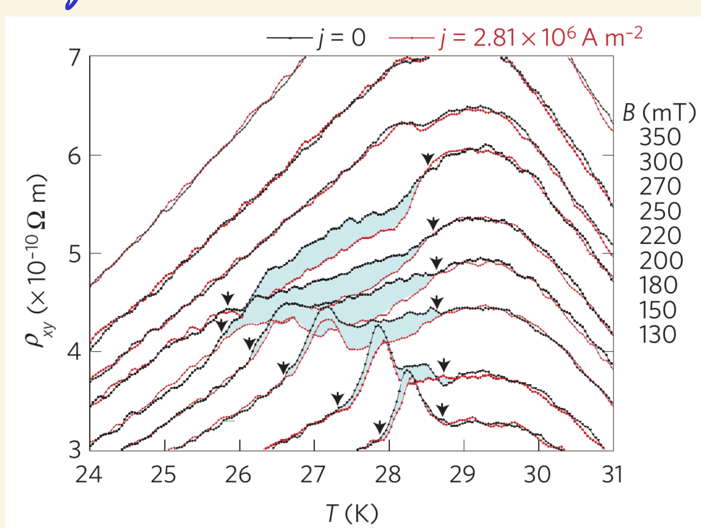
$\Rightarrow$ spin motive force

voltage from magnetization dynamics



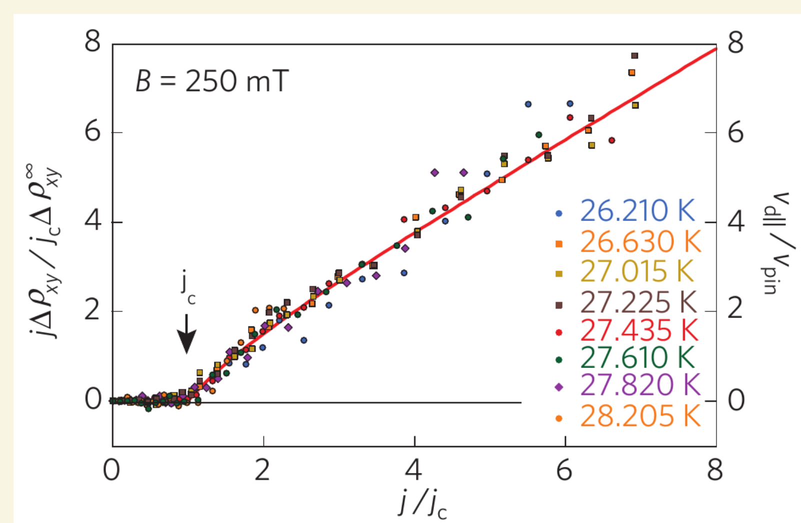
Skyrmion   Schulz, Nat. Phys. (2013)

- (topological) Hall effect B_s



- Spin motive force $E_s \propto v$

skyrmion velocity $\propto$ applied current j



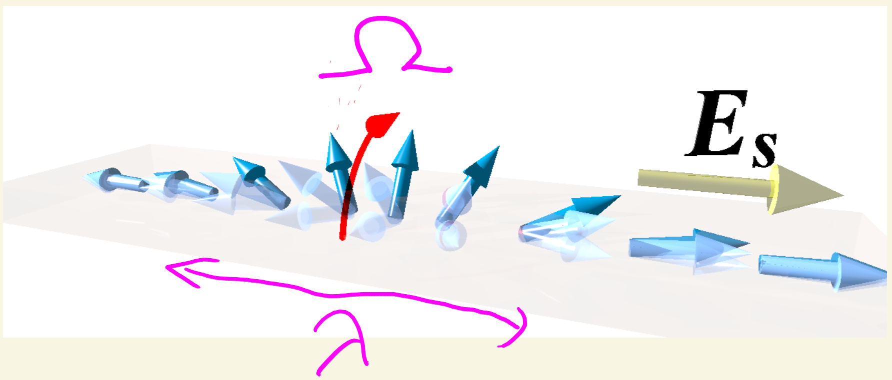
Theoretical values

- in the electromagnetism unit

$$E_{s,i} = \frac{\hbar}{2e}\, \mathbf{n}\cdot(\dot{\mathbf{n}} \times \nabla_i \mathbf{n}) \sim \underbrace{\frac{\hbar}{2e}}\, \Omega/\lambda$$

Ω : frequency of spin dynamics

λ : length scale of spin structur



$\frac{\hbar}{2e} = 3.4 \times 10^{-16}\ \mathrm{V \cdot s} = \mathrm{Tm}^2$

$\Omega = 100\,\mathrm{GHz} = 10^{8}\,\mathrm{Hz}$

$\Rightarrow E_s \lambda = 3.4 \times 10^{-8}\ \mathrm{V}$ for a DW

$\lambda = 10\,\mathrm{nm} = 10^{-8}\,\mathrm{m}$

$\Rightarrow E_s = 3.4\ \mathrm{V/m}$

$$B_{s,i} = \frac{\hbar}{4e}\, \varepsilon_{ijk}\, \mathbf{n}\cdot(\partial_j \mathbf{n} \times \partial_k \mathbf{n}) \sim \frac{\hbar}{2e}\frac{1}{\lambda^2}$$

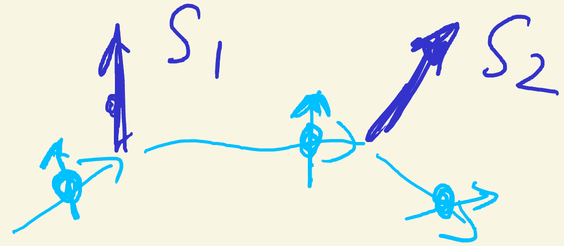
$\lambda = 10\,\mathrm{nm} \Rightarrow B_s = 3.4\,\mathrm{T}$

$\lambda = 1\,\mathrm{nm}$ $340\,\mathrm{T}$!

Nature is much stronger than human technology

Origin of gauge field   spin commutation relation

· sd exchange interaction   perturbative

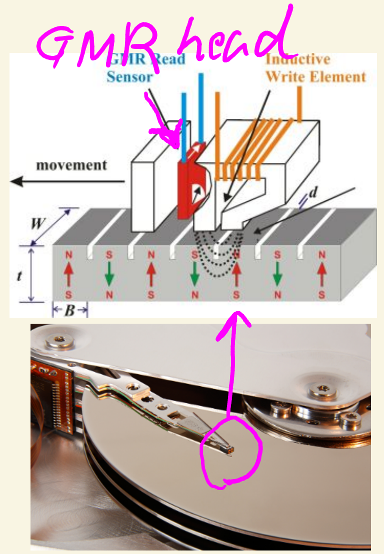


scattering amp $\propto J\, \vec{S}_i \cdot \vec{\sigma}$ conduction electron spin

· 2nd order amplitude

$$A_2 = J^2 (\vec{S}_1 \cdot \vec{\sigma})(\vec{S}_2 \cdot \vec{\sigma})$$

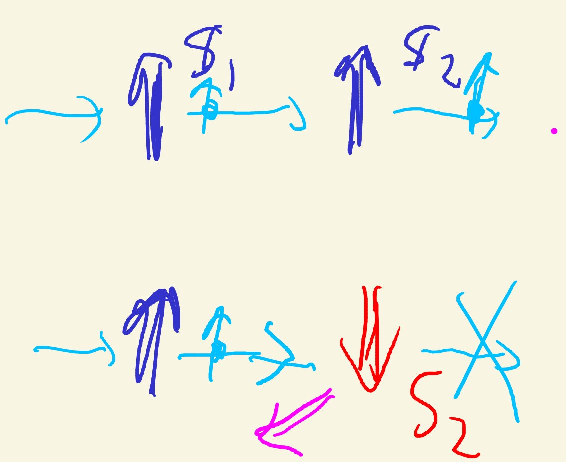
$$= J^2 \left[\vec{S}_1 \cdot \vec{S}_2 + i (\vec{S}_1 \times \vec{S}_2) \cdot \vec{\sigma} \right]$$

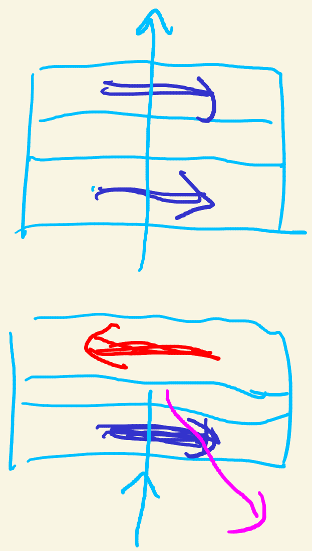


charge part   tr in spin index

$$\mathrm{tr}\, A_2 = 2J^2\, \vec{S}_1 \cdot \vec{S}_2$$

resistance due to spin mismatch





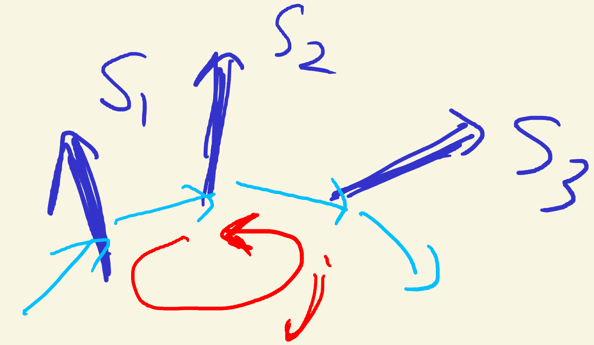
GMR giant magneto-resistance

Nobel prize 2007

A. Fert, P. Grünberg

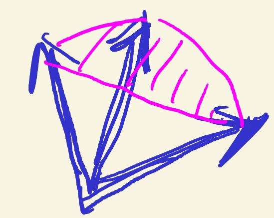
3rd order

$$A_3 = J^3 (\boldsymbol{S}_1\cdot\sigma)(\boldsymbol{S}_2\cdot\sigma)(\boldsymbol{S}_3\cdot\sigma)$$



charge part $\quad \mathrm{tr}[\sigma_i \sigma_j \sigma_k] = 2i\varepsilon_{ijk}$

$$\mathrm{tr}\, A_3 = 2J^3 i \boxed{\boldsymbol{S}_1\cdot(\boldsymbol{S}_2\times\boldsymbol{S}_3)} \equiv C_{123}$$



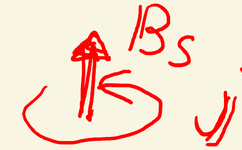
non-coplanarity

solid angle

Time reversal broken

$\Rightarrow$ emergent rotational current

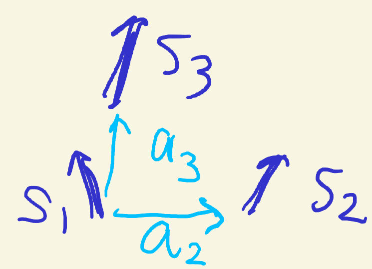
$$j \propto C_{123}$$



GT & Kohno 2003 PRB

$\Rightarrow$ emergent (= effective) magnetic field $\quad B_s \propto C_{123}$

continuum limit



$$\boldsymbol{S}_2 = \boldsymbol{S} + (a_2\cdot\nabla)\boldsymbol{S} + \cdots$$
$$\boldsymbol{S}_1 = \boldsymbol{S}$$
$$\boldsymbol{S}_3 = \boldsymbol{S} + (a_3\cdot\nabla)\boldsymbol{S} + \cdots$$

$$C_{123} = a_2^i a_3^j\, \boldsymbol{S}\cdot(\nabla_i \boldsymbol{S}\times\nabla_j \boldsymbol{S})$$

Spin Berry phase B_S

Spin Berry phase is due to spin commutation relation

$$[\sigma_i, \sigma_j] = i\varepsilon_{ijk}\sigma_k$$

Spin part

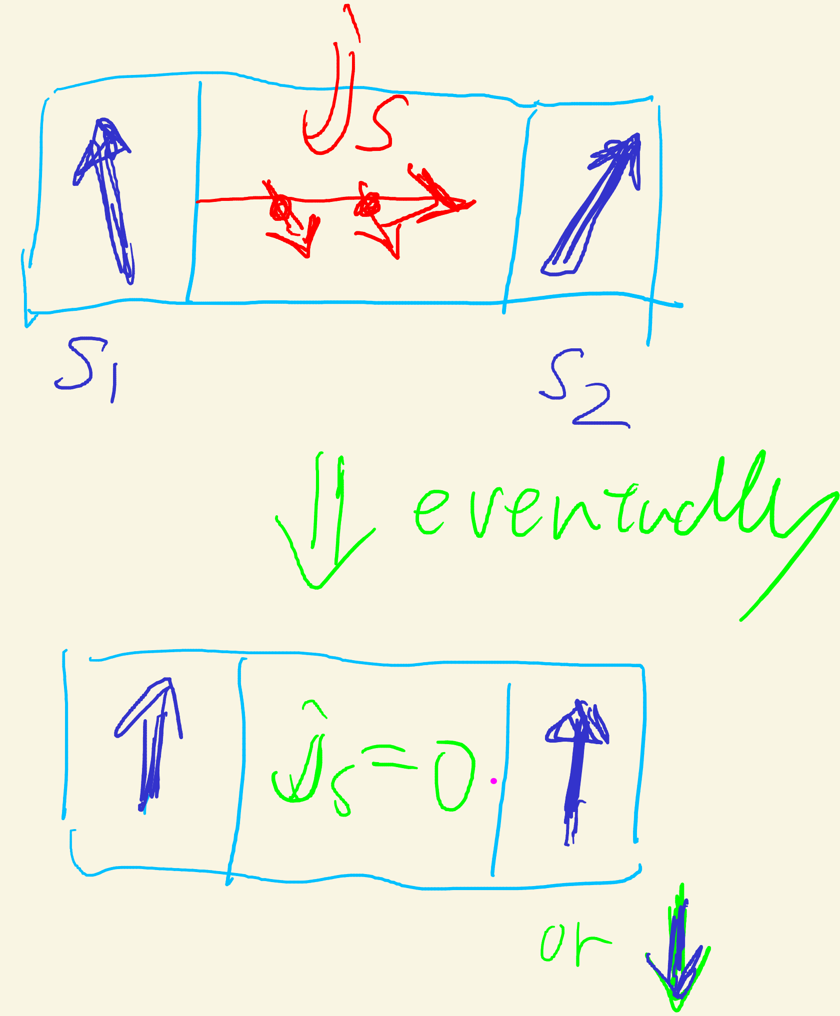
2nd order $A_2 = J^2(\boldsymbol{S}_1\cdot\boldsymbol{\sigma})(\boldsymbol{S}_2\cdot\boldsymbol{\sigma})$

$$\mathrm{tr}[\boldsymbol{\sigma} A_2] = 2J^2(\boldsymbol{S}_1 \times \boldsymbol{S}_2)$$

Spin current j_s

|| equivalent to

torque



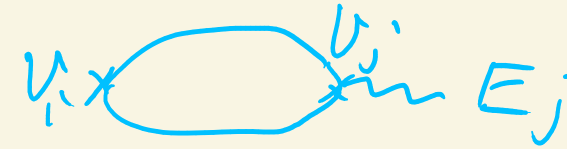
Berry phase in momentum space

linear response theory   Hall conductivity

$$\sigma_{ij} = \lim_{\Omega\to 0}\frac{1}{\Omega}\sum_{k\omega} \mathrm{tr}\left[V_i\, G_{k\omega}\, V_j\, G_{k,\omega+\Omega}\right]$$

lesser Green's function

$$V_i = \frac{\partial}{\partial k_i}\varepsilon_k = -\frac{\partial}{\partial k_i}\left(G^{-1}_{k\omega}\right)$$



wave-function representation

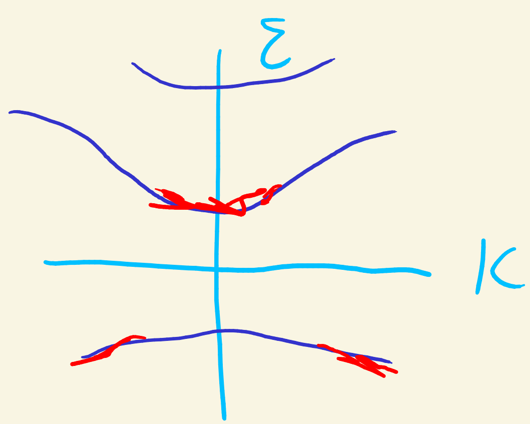
$$\sigma_{ij} = \frac{e^2}{h}\int dk\, f(k)\,\Omega_{ij}(k)$$

$$\Omega_{ij} = \partial_{k_i} A_j(k) - \partial_{k_j} A_i(k)$$

Berry curvature in k-space

$$A_i(k) = -i\langle k|\partial_{k_i}|k\rangle$$

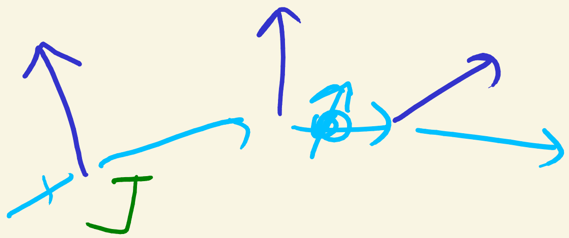
gauge field



$\Omega_{ij}(k)$ distribution

2 Berry phases

- Real Space Berry phase



$S_1\cdot(S_2\times S_3)$

dirty limit

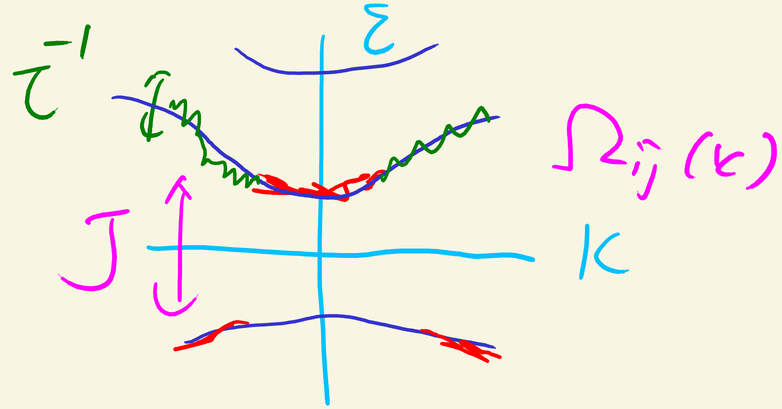
$J\tau < 1$

τ: relaxation time

τ^{-1}

band picture not good

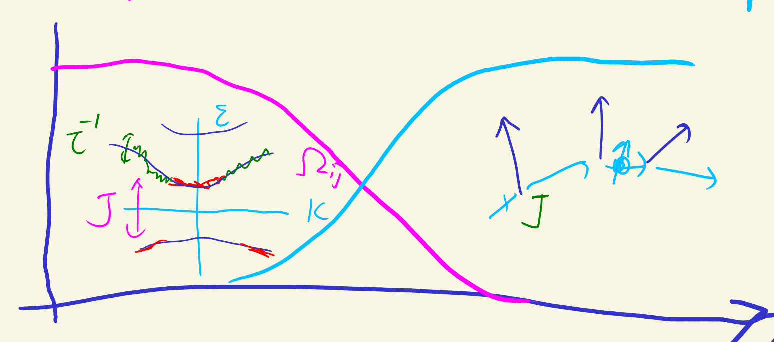
- Momentum Space Berry phase



clean limit

$J\tau > 1$

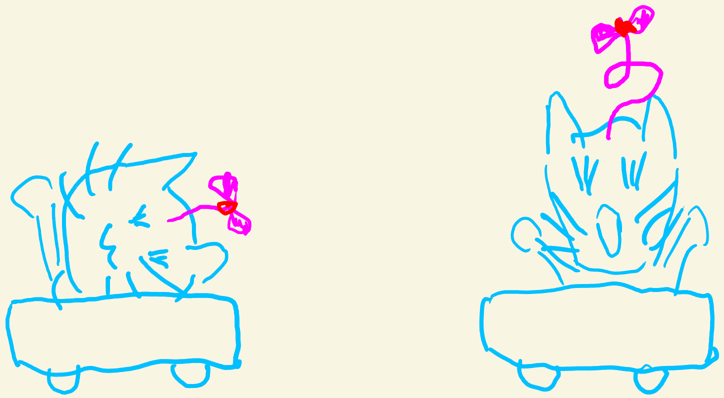
k-space   real space



Onoda GT Nagaosa J Phys. Soc. Jpn 2004

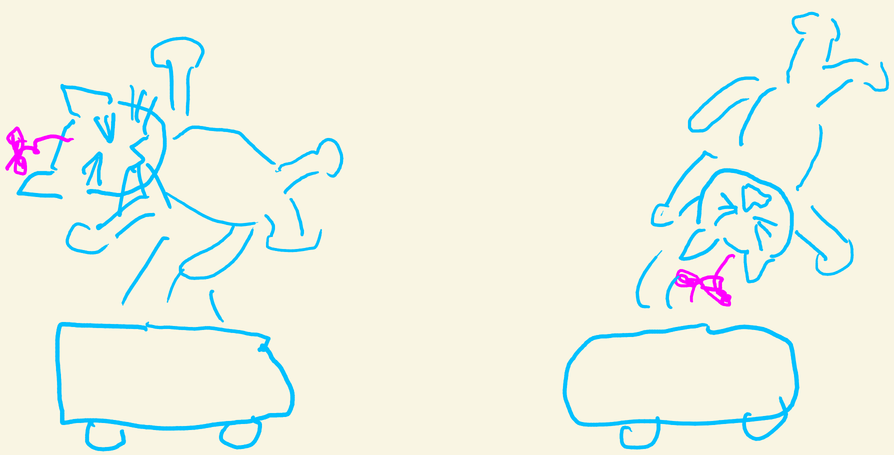
Non adiabaticity

Adiabatic



stays in the ground state

Non adiabatic



excited states

Effective gauge field including non-adiabaticity

- $A_\mu = -i U^{-1} \partial_\mu U$

$$= \frac{1}{2}\Big[(-\partial_\mu\theta \sin\phi - \sin\theta\cos\phi\,\partial_\mu\phi)\sigma_x + (\partial_\mu\theta\cos\phi - \sin\theta\sin\phi\,\partial_\mu\phi)\sigma_y + \boxed{(1-\cos\theta)\partial_\mu\phi\,\sigma_z}\Big]$$

electromagnetism

$$= \frac{1}{2}\begin{bmatrix} \boxed{(1-\cos\theta)\partial_\mu\phi} & \boxed{(-i\partial_\mu\theta - \sin\theta\,\partial_\mu\phi)e^{-i\phi}} \\ \boxed{(i\partial_\mu\theta - \sin\theta\,\partial_\mu\phi)e^{i\phi}} & \boxed{-(1-\cos\theta)\partial_\mu\phi} \end{bmatrix}$$

adiabatic

non-adiabatic

electron spin flip

- gauge coupling to spin current

$$\boxed{H_A = -A_\mu^\alpha j_{s\mu}^\alpha + O(A^2)}$$

$j_{si}^\alpha = \frac{p_i}{m}\sigma_\alpha$ spin current

$j_{st}^\alpha = \sigma_\alpha$ spin density

$$A_\mu^\alpha = \frac{1}{2}\begin{pmatrix} -\partial_\mu\theta\sin\phi - \sin\theta\cos\phi\,\partial_\mu\phi \\ \partial_\mu\theta\cos\phi - \sin\theta\sin\phi\,\partial_\mu\phi \\ (1-\cos\theta)\partial_\mu\phi \end{pmatrix}$$

3 component (SU(2)) gauge field

$$= \underbrace{\frac{1}{2}\, n \times \partial_\mu n}_{\text{non-adiabatic}} - \underbrace{A_\mu^z\, n}_{\text{adiabatic component}}$$

Some effects arizing from gauge coupling

- Spin-transfer effect
- Dzyaloshinskii-Moriya interaction

$$H_A = -A^{\alpha}_{\mu} \cdot j^{\alpha}_{s\mu}$$

1. Spin-transfer effect adiabatic limit

$j^{\alpha}_{s\mu} = \delta_{\alpha z}\, j_{s\mu}$ spin-polarization // localized spin

$$\Rightarrow \quad H_A = -A_{s\mu} \cdot j_{s\mu} \qquad A_{s\mu} = \frac{1}{2}(1-\cos\theta)\,\partial_\mu \phi$$

☆ Represents

- effects of localized spin (θ, ϕ) on electrons
  voltage generation, Hall effect (E_s, B_s)
- effects on localized spin
  electron spin current j_s induces a torque on (θ, ϕ)

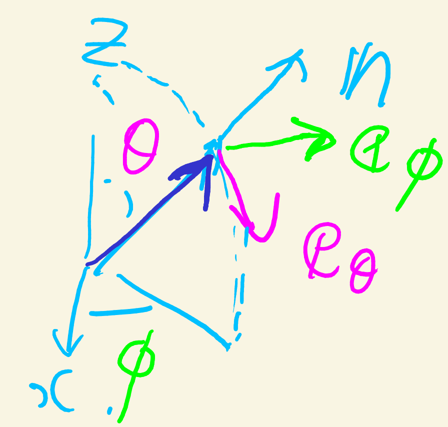
$$H_A^{(ad)} = -\frac{1}{2}(1-\cos\theta)(\underset{\sim}{\mathbb{J}_s}\cdot\nabla)\phi$$

Spin current (intrinsic or extrinsic)

- strange form representing $n = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
  - geometrical meaning
- calculate torque

If magnetic field $\quad H_B = -B\cdot S \Rightarrow \quad T = B\times S$

$$B = -\frac{\delta H_B}{\delta S}$$



$$B_A = -\frac{\delta H_A^{(ad)}}{\delta S} = ?$$

$$-\frac{\delta H}{\delta S} = -\left(\frac{\delta\theta}{\delta S}\frac{\delta H}{\delta\theta} + \frac{\delta\phi}{\delta S}\frac{\delta H}{\delta\phi}\right)$$

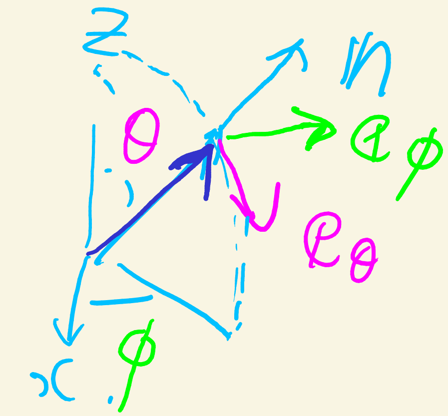
$$\frac{\delta\theta}{\delta S} = \frac{1}{S}(\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) = \frac{1}{S}e_\theta$$

$$\frac{\delta\phi}{\delta S} = \frac{1}{S}\frac{1}{\sin\theta}(-\sin\phi, \cos\phi, 0) = \frac{1}{S}\frac{1}{\sin\theta}e_\phi$$

$$-\frac{\delta H_A^{ad}}{\delta\theta} = \sin\theta(j_s\cdot\nabla)\phi \qquad -\frac{\delta H_A^{ad}}{\delta\phi} \underset{\text{"partial derivative"}}{=} -\frac{1}{2}(j_s\cdot\nabla)\cos\theta$$

$$-\frac{\delta H_A^{ad}}{\delta S} = \frac{1}{2}\left[e_\theta\sin\theta(j_s\cdot\nabla)\phi - e_\phi(\mathbb{J}_s\cdot\nabla)\theta\right]$$

$$T_A = -\frac{\delta H_A^{ad}}{\delta S} \times S \qquad S = S n$$



$$= \frac{1}{2}\Big(\underbrace{(e_\theta \times n)}_{-e_\phi} \sin\theta (j_s \cdot \nabla)\phi - \underbrace{(e_\phi \times n)}_{e_\theta} (j_s \cdot \nabla)\theta \Big)$$

$$= -\frac{1}{2} j_{si} \left(e_\phi \sin\theta \nabla_i \phi + e_\theta \nabla_i \theta \right)$$

$$= -\frac{1}{2} j_{si} \nabla_i n \qquad = -\frac{1}{2} (j_s \cdot \nabla) n$$

- Spin current induces inhomogeneity of n

$$\uparrow \uparrow \xrightarrow{j_s} \uparrow \uparrow \quad \Rightarrow \quad \nwarrow \uparrow \nearrow \rightarrow$$

sd exchange interaction between electron spin and localized spin

- What is the configuration of n ? under j_s

$\Rightarrow$ study dynamics!

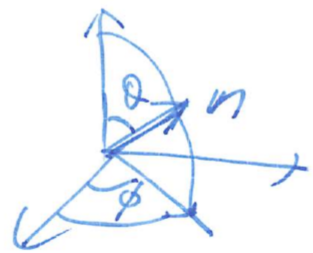
Derivation of $B_A(r) = -\frac{\delta H_A^{(ad)}}{\delta n(r)}$

$$H_A^{(ad)} = \int \left[-\frac{1}{2}(1-\cos\theta)(J_s\cdot\nabla)\phi \right] dr$$

$$\left(= \int \left[\frac{1}{2}\left[(J_s\cdot\nabla)(1-\cos\theta)\right] \phi \right] dr \right).$$

$$n(r) = (\overset{n_x}{\sin\theta\cos\phi}, \overset{n_y}{\sin\theta\sin\phi}, \overset{n_z}{\cos\theta})$$

$\theta(r)$, $\phi(r)$



$$\frac{\delta H(\theta,\phi)}{\delta n} = \underbrace{\frac{\delta\theta}{\delta n}}_{\parallel}\frac{\delta H}{\delta\theta} + \frac{\delta\phi}{\delta n}\frac{\delta H}{\delta\phi}$$

• variation of θ when n_i is changed fixing other n_j's

$$\frac{\delta\theta}{\delta n} = \left(\frac{\delta\theta}{\delta n_x}, \frac{\delta\theta}{\delta n_y}, \frac{\delta\theta}{\delta n_z} \right)$$

$\theta(n_x, n_y, n_z)$: $\quad \tan\theta = \frac{\sqrt{n_x^2+n_y^2}}{n_z}$

$$\frac{\delta\theta}{\delta n_x} = \frac{\delta\tan\theta}{\delta n_x}\frac{\delta\theta}{\delta\tan\theta} = \frac{d}{dn_x}\frac{\sqrt{n_x^2+n_y^2}}{n_z}\cdot\frac{1}{\frac{d}{d\theta}\tan\theta}$$

$$= \frac{n_x}{n_z\sqrt{n_x^2+n_y^2}}\frac{1}{\frac{1}{\cos^2\theta}} = \cos\theta\cos\phi$$

$$\frac{\delta\theta}{\delta n_y} = \cos\theta\sin\phi$$

$$\frac{\delta\theta}{\delta n_z} = \frac{d}{dn_z}\frac{\sqrt{n_x^2+n_y^2}}{n_z}\cos^2\theta = -\sin\theta.$$

$$\frac{\delta\theta}{\delta n} = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) = e_\theta$$

$$\frac{\delta\phi}{\delta n} = \frac{\delta\tan\phi}{\delta n}\frac{\delta\phi}{\delta\tan\phi} = \left(-\frac{n_y}{n_x^2}, \frac{1}{n_x}, 0\right)\cos^2\phi$$

$\tan\phi = \frac{n_y}{n_x}$

$$= \frac{1}{\sin\theta}(-\sin\phi, \cos\phi, 0)$$

$$= \frac{1}{\sin\theta} e_\phi$$

$$\frac{\delta H_A^{(ad)}}{\delta\theta} = -\frac{1}{2}\sin\theta(J_s\cdot\nabla)\phi$$

$$\frac{\delta H_A^{(ad)}}{\delta\phi} = -\frac{1}{2}(J_s\cdot\nabla)\cos\theta = \frac{1}{2}\sin\theta(J_s\cdot\nabla)\theta.$$

$$\Rightarrow \boxed{B_A = -\frac{\delta H_A^{ad}}{\delta n} = \frac{1}{2}\left[\sin\theta(-e_\theta)(J_s\cdot\nabla)\phi + e_\phi(J_s\cdot\nabla)\theta\right]}$$

$$\boxed{= \frac{1}{2} n \times (J_s\cdot\nabla) n}$$

$$\nabla_i n = (\nabla_i\theta)e_\theta + \sin\theta(\nabla_i\phi)e_\phi$$

$$n\times\nabla_i n = (\nabla_i\theta)\underbrace{(n\times e_\theta)}_{\overset{\parallel}{e_\phi}} + \sin\theta(\nabla_i\phi)\underbrace{(n\times e_\phi)}_{-e_\theta}$$

$$= (\nabla_i\theta)e_\phi - \sin\theta(\nabla_i\phi)e_\theta$$

Spin dynamics

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{B} \times \mathbf{S} \qquad \mathbf{B} = -\frac{\delta H}{\delta \mathbf{S}}$$ total magnetic field

$-\gamma = -\frac{e}{m} \to 1$

adiabatic gauge field

$$\mathbf{B}_A = -\frac{1}{2S} j_{si} (\mathbf{n} \times \nabla_i \mathbf{n}) \qquad \mathbf{B}_A \times \mathbf{S} = -\frac{1}{2} j_{si} \nabla_i \mathbf{n}$$

$$\Rightarrow \frac{\partial \mathbf{S}}{\partial t} = \mathbf{B}_A \times \mathbf{S} \Rightarrow \partial_t \mathbf{n} = -\frac{1}{2S} (\mathbf{j}_s \cdot \nabla) \mathbf{n}$$

$$\boxed{\left[\frac{\partial}{\partial t} + \frac{1}{2S}(\mathbf{j}_s \cdot \nabla)\right] \mathbf{n}(\mathbf{r}, t) = 0}$$

Galilean invariant

$\mathbf{n}(\mathbf{r}, t)$ flows with $\mathbf{j}_s$

Solution (general)

$\mathbf{n}(\mathbf{r} - \mathbf{v}_s t)$

$$\boxed{\mathbf{v}_s = \frac{1}{2S} \mathbf{j}_s} \qquad \left(\frac{a^3}{2S} \mathbf{j}_s\right)$$

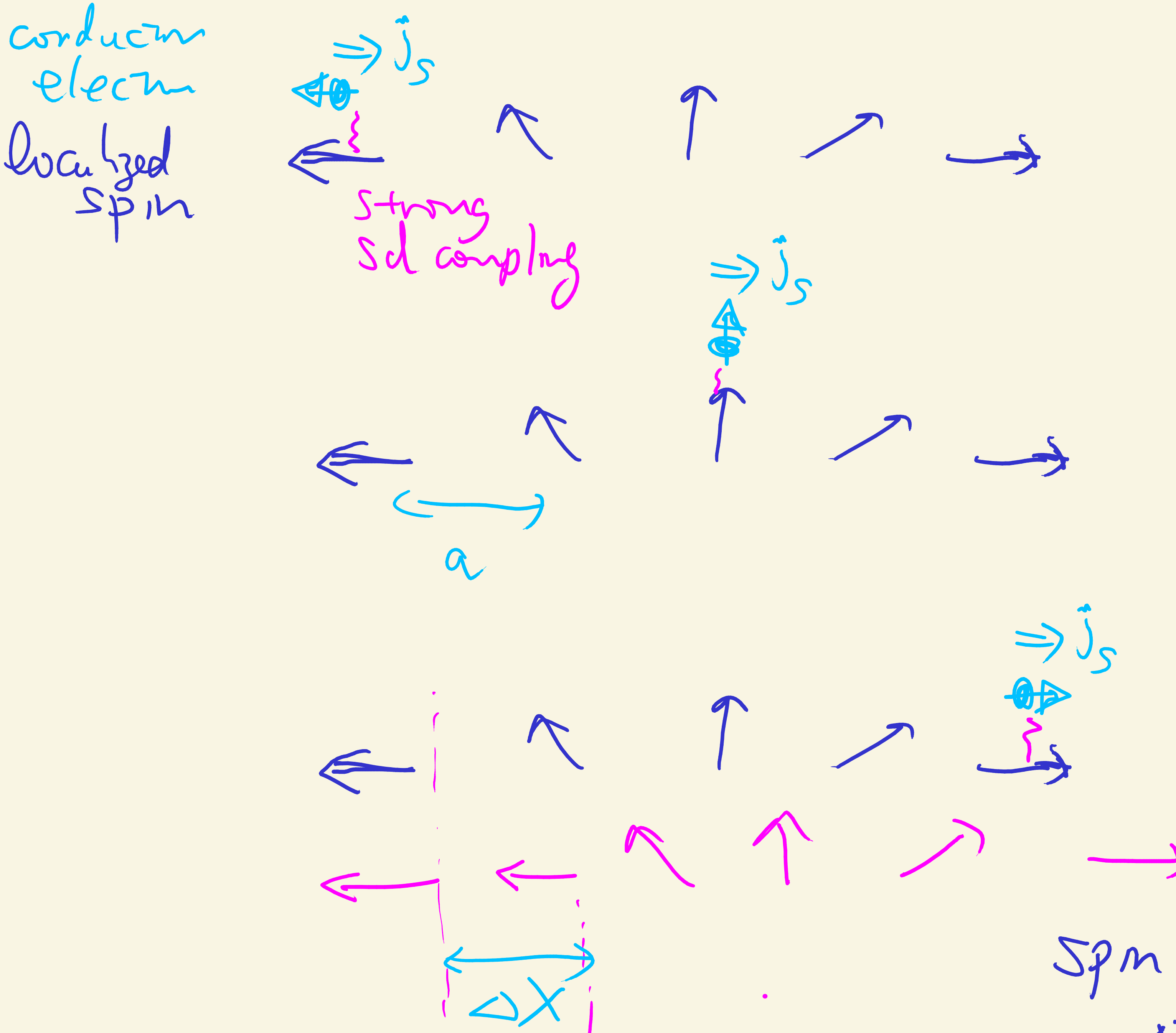
$m^3 \cdot m/(s\, m^2)$

Spin current (adiabatic) pushes any magnetization structure to move at $\mathbf{v}_s$

Spin-transfer effect

Physical mechanism of spin transfer

• a domain wall



injected electron spin is reversed finally

$\Downarrow$

spin angular momentum increase of $\frac{\hbar}{2} \times 2$

$\Downarrow$

localized spins compensate for it

$\Downarrow$

spin structure moves

$$\Delta X = \frac{a}{2S}$$

lattice const

· continuous spin injection (spin current)

$\dot{j}_s = vn$ spin current density $\frac{m}{s} \cdot \frac{1}{m^3} = \frac{1}{m^2 s}$

$\uparrow$ $\nwarrow$ electron density

Electron velocity (Fermi velocity)

without spin $\frac{1}{2}$.

$\Rightarrow$ $vna^2 = \dot{j}_s a^2$ electrons injected / sec.

$\Rightarrow$ domain wall velocity

$v_w = \Delta X \cdot \dot{j}_s a^2 = \frac{a^3}{2S} \dot{j}_s$ L. Berger 1986

· transfer of spin angular momentum between localized spin and conduction electron

· universal for any spin configuration

The 2-sheet explanation is summarized in the gauge coupling

$H_A = -\frac{1}{2}(1-\cos\theta)(\dot{j}_s \cdot \nabla)\phi$

Even simpler in Lagrangian $L = -S(1-\cos\theta)\dot{\phi} - H$

$L = -S(1-\cos\theta)(\partial t - \frac{1}{2S}\dot{j}_s \cdot \nabla)\phi$

Exercise

Show that a Lagrangian

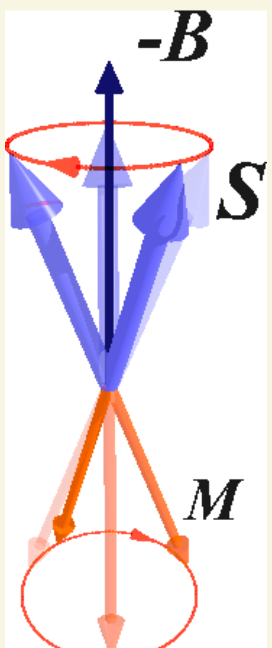
$$\mathcal{L} = -S(1-\cos\theta)\partial_t\phi - S\mathbb{B}\cdot n \qquad n = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

leads to an equation of motion

$$\partial_t \mathbb{n} = \mathbb{B}\times n$$

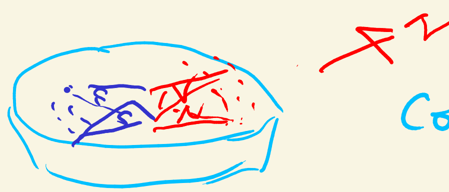
Landau-Lifshitz equation (LL)

LL equation is not realistic $\Rightarrow$ Landau-Lifshitz-Gilbert (LLG) equation

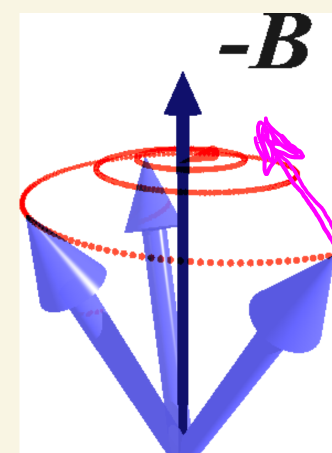


- Spin keep precessing
- never points the direction of $-\mathbb{B}$

$$\partial_t \mathbb{n} = \mathbb{B}\times\mathbb{n} - \alpha\,\mathbb{n}\times\partial_t\mathbb{n}$$



Compass does not work



damping torque

$\alpha \sim 0.01$

magnetization

$$M = \mu_0 \frac{\hbar\gamma}{a^3}\mathbb{S}$$

electron charge

$$\gamma = \frac{e}{m} < 0$$

Gilbert damping ⇒ Out of plane motion is essential

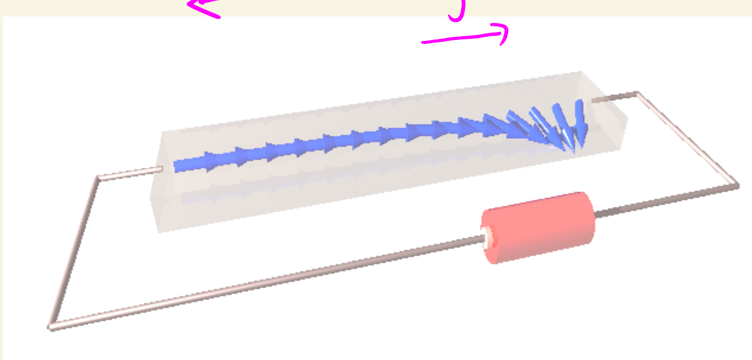
spin transfer effect with damping

$$\underbrace{(\partial_t + \mathbf{v}_s \cdot \nabla)\mathbf{n}}_{\text{sliding motion}} = -\alpha\, \underbrace{\mathbf{n} \times \partial_t \mathbf{n}}_{\text{out-of plane torque}}$$

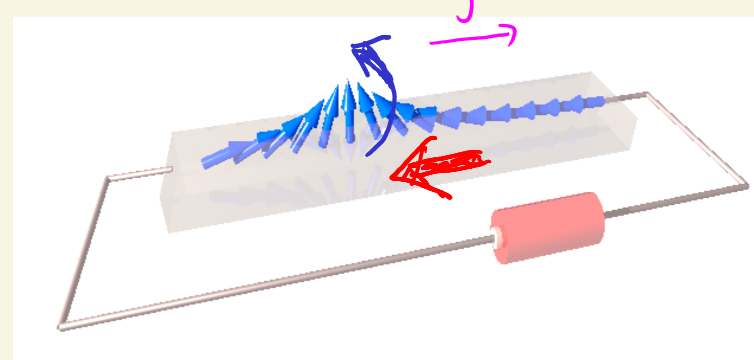
Simple sliding is not possible

Domain wall under applied spin current and damping
(spin transfer)

$v_{electron}$ ← j →



j →



Sliding + rotation

spin transfer damping

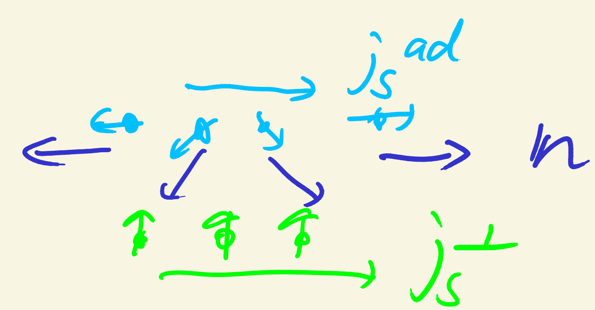
Some effects arising from gauge coupling

✓ Spin-transfer effect adiabatic limit

⇒ ⊙ Dzyaloshinskii-Moriya interaction nonadiabaticity

$$H_A = -A_\mu^\alpha \cdot j_{s\mu}^\alpha$$

$$j_{s\mu}^\alpha = \underset{/\!/ n}{j_{s\mu}^{ad}} + \underset{\perp n}{j_{s\mu}^{\perp}}$$



$$A_\mu^\alpha = \frac{1}{2}\begin{pmatrix} -\partial_\mu\theta \sin\phi - \sin\theta\cos\phi\,\partial_\mu\phi \\ \partial_\mu\theta\cos\phi - \sin\theta\sin\phi\,\partial_\mu\phi \\ (1-\cos\theta)\partial_\mu\phi \end{pmatrix}$$

$$= \underbrace{\frac{1}{2}\, n \times \partial_\mu n}_{\text{nonadiabatic}} - A_\mu^z\, n$$

non-adiabatic contribution

$$\boxed{H_A^{na} = -A_\mu^\perp j_{s\mu}^\perp = -\frac{1}{2}\, j_{s\mu}^\perp \cdot (n \times \partial_\mu n)}$$

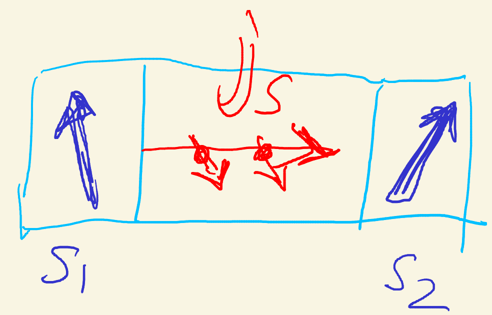
$$\boxed{H_A^{na} = -A_\mu^\perp j_{s\mu}^\perp = -\frac{1}{2} j_{s\mu}^\perp \cdot (\mathbf{n} \times \partial_\mu \mathbf{n})}$$

- spin $\mathbf{n}$ $\Rightarrow$ electron

  spin current generation

$$j_s^\perp \propto \mathbf{n} \times \nabla \mathbf{n}$$

$$s^\perp \propto \mathbf{n} \times \partial_t \mathbf{n}$$



- electron spin current $\Rightarrow$ $\mathbf{n}$

$$H_A^{na} = -\mathbf{D}_\mu \cdot \underline{(\mathbf{n} \times \nabla_\mu \mathbf{n})} \qquad \mathbf{D}_\mu = \frac{1}{2} j_{s\mu}^\perp$$

twist spin structure

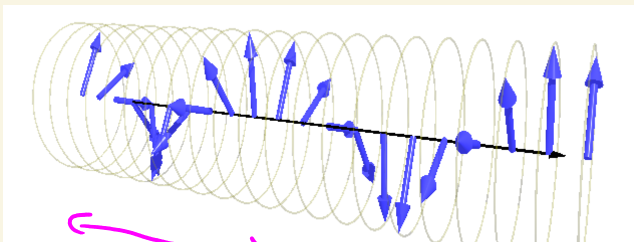
Dzyaloshinskii - Moriya interaction

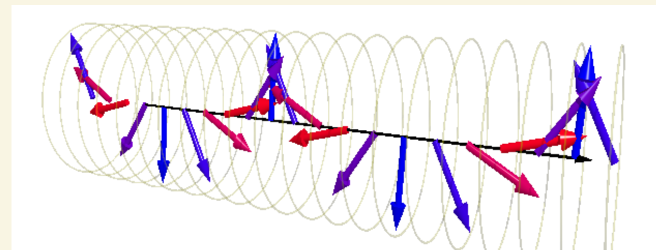
Dzyaloshinskii-Moriya interaction

$$H_A^{na} = -D_\mu \cdot (n \times \nabla_\mu n)$$

ferromagnetic interaction

$$H_J = \frac{J}{2}(\nabla n)^2$$

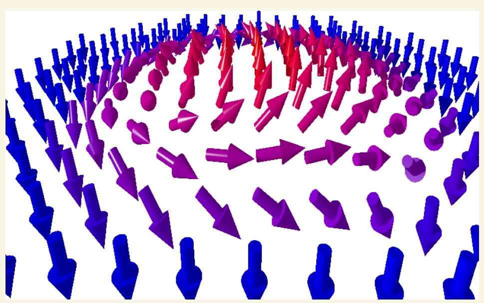




Spiral structures

period

$\lambda = J/D$



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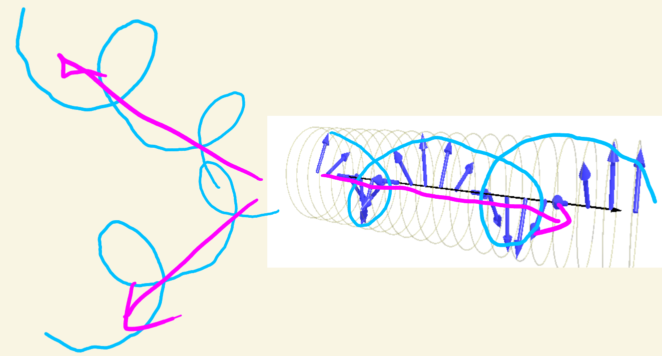
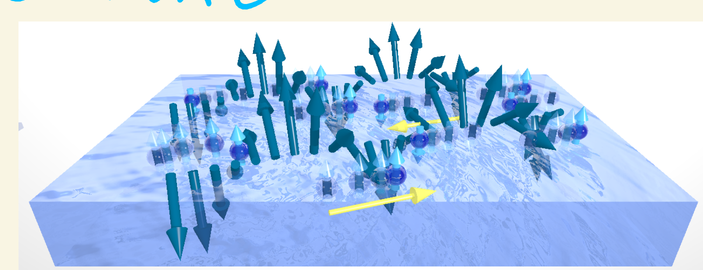


Superposition of 3 helix

Skyrmion

$D \propto j_s^{\perp}$: DM interaction and spiral structures arize from intrinsic spin current

Doppler shift of spin



Prediction of DM constant

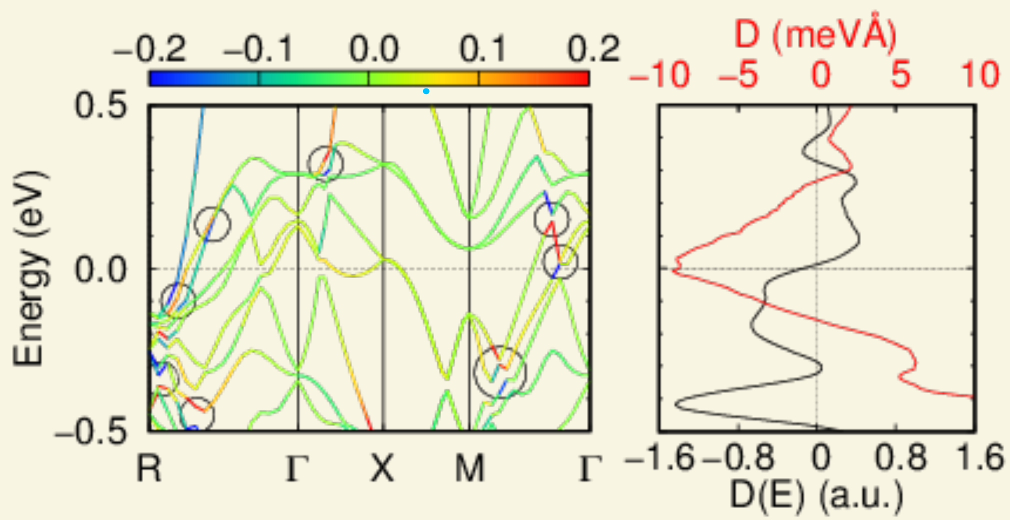
$D_\mu = \frac{1}{2} j^{\perp}_{s\mu}$ evaluate intrinsic spincurrent ← broken inversion symmetry

Spin-orbit interaction

$j_s \xrightarrow{P} -j_s$ · space inversion $(x, y, z) \to (-x, -y, -z)$

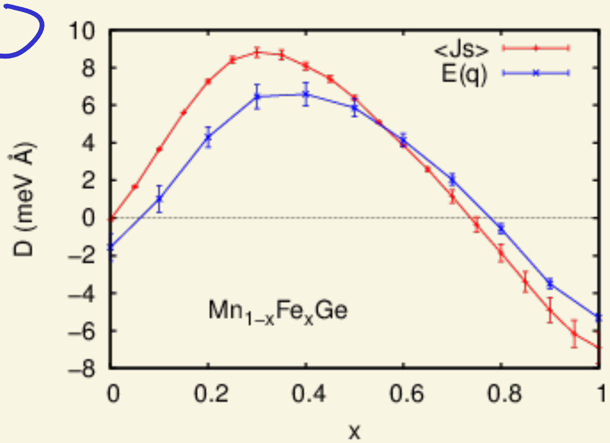
$j_s \xrightarrow{T} j_s$ · time reversal $t \to -t$

First principles calculation   Kikuchi, GT, PRL 2016



Spincurrent distribution of FeGe

D



Practical evaluation scheme

(conventional theory "Berry phase" representation)

heavy calculation

Spin orbit interaction

$$H_{so} = \frac{\hbar}{4m^2c^2}(\nabla V \times \mathbf{p})\cdot\boldsymbol{\sigma}$$

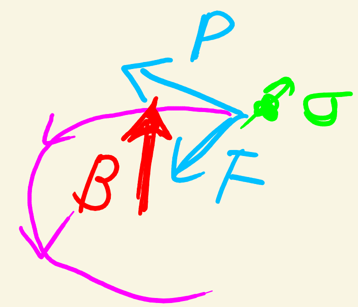
- relativistic correction in Dirac equation $\propto \frac{1}{c^2}$
- V: any potential
- couples orbital motion $\mathbf{p}$ and electron spin $\boldsymbol{\sigma}$

- origin

$$\nabla V \times \mathbf{p} = -(\mathbf{F} \times \mathbf{p}) \qquad \mathbf{F} = -\nabla V$$

$$\simeq \mathbf{B} \quad \Rightarrow (\nabla V \times \mathbf{p})\cdot\boldsymbol{\sigma} \sim \mathbf{B}\cdot\boldsymbol{\sigma}$$

rotational motion



- spherical potential (Coulomb etc)

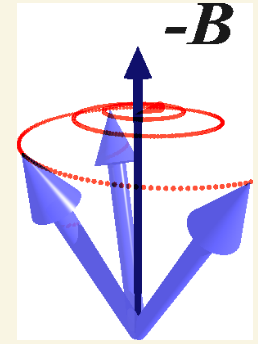
$$\nabla V = \hat{\mathbf{r}}\,\partial_r V(r) \Rightarrow \nabla V \times \mathbf{p} = \mathbf{l}\,\frac{1}{r}\partial_r V(r) \qquad \mathbf{l} = \mathbf{r} \times \mathbf{p}$$

orbital angular momentum

$$\Rightarrow H_{so} = \lambda_{so}\,\mathbf{l}\cdot\boldsymbol{\sigma} \qquad \text{LS coupling} \qquad \lambda_{so} = \frac{\hbar^2}{4m^2c^2}\left\langle \frac{1}{r}\partial_r V \right\rangle$$

Spin-orbit interaction

$$H_{so} = \frac{\hbar}{4m^2c^2}(\nabla V \times \mathbf{P}) \cdot \sigma$$



sd int

localized spin $\mathbf{S}$ ⇒ electron spin σ

⇓ so int

lattice (phonon)

- Causes spin relaxation

  Gilbert damping $\alpha \propto \lambda_{so}^2$

- Couples electron orbital motion and spin

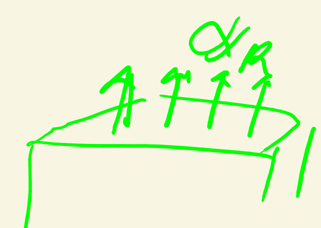
  useful for spintronics

  $M \propto \mathbf{S} = \kappa \mathbf{E}$ cross-correlation

  mixing $\mathbf{E}$ and $\mathbf{B}$ (M)

- approximated by an effective gauge field

  $H_{so} = \mathbf{k} \cdot A_{so}$ $\quad A_{so} = \frac{-\hbar^2}{4m^2c^2}(\nabla V \times \sigma)$



inversion symmetry breaking ⇒ $-\nabla V$ = const

surface, interface $\quad \alpha_R = \frac{-\hbar^2 \nabla V}{4m^2c^2}$

⇒ $A_{so} = \alpha_R \times \sigma$ $\quad$ Rashba model

Rashba model

$$H_R = -\mathbb{k}\cdot\mathbb{A}_R \qquad \mathbb{A}_R = \underline{\alpha_R} \times \sigma$$

$$= \alpha_R\cdot(\mathbb{k}\times\sigma)$$

Rashba field

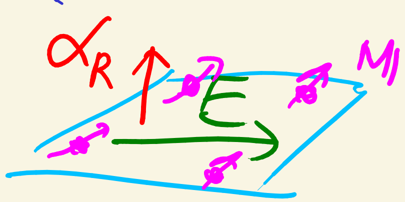
∥ z axis

- originally 2D electron gas semiconductor
- now
  - most metallic surface Au, Ag
  - with impurities Bi
  - bulk system BiTeI

• $E \Rightarrow j \propto \mathbb{k} \Rightarrow \sigma$

$$M = \kappa_{ME}(\alpha_R \times E)$$

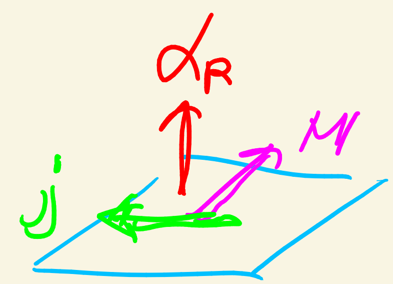
Rashba-Edelstein effect



• $B \Rightarrow j$

$$j = \kappa_{ME}(\alpha_R \times \dot{B})$$

$\sim \partial_t A_R$



Effective electromagnetic field

$$E_R = -\dot{A}_R$$

$$= \alpha_R \times \dot{n}$$

$$B_R = \nabla\times(\alpha_R \times n)$$

Voltage generation from $\dot{n}$

B-E conversion in a plane $\perp \alpha_R$

Effective gauge field

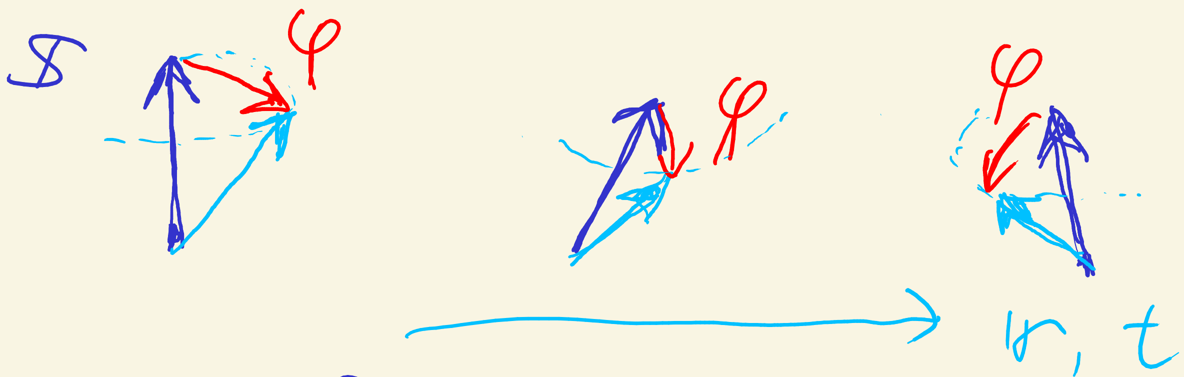
applies to anyone on a cart

coupled to magnetization structure

strongly

- ✓ conduction electron
- → spin wave (magnon)
- phonon
- → photon (light)

Spin wave



$\varphi(\mathbf{r}, t)$ magnon (spin wave) field

· uniform $\mathbf{S}$ ferromagnet

$$\mathbf{S} \sim \begin{pmatrix} \varphi_x \\ \varphi_y \\ S \end{pmatrix} + O(\varphi^2)$$

· quantum mechanical commutation relation

$$[\hat{S}_x, \hat{S}_y] = i\hat{S}_z \Rightarrow [\hat{\varphi}_x, \hat{\varphi}_y] = iS$$

$$\Updownarrow$$

$$[a, a^+] = 1$$ boson commutation relation

$$\Rightarrow \quad \hat{\varphi}_x = \frac{1}{\sqrt{2S}}(\hat{a} + \hat{a}^+)$$

$$\hat{\varphi}_y = \frac{-i}{\sqrt{2S}}(\hat{a} - \hat{a}^+)$$

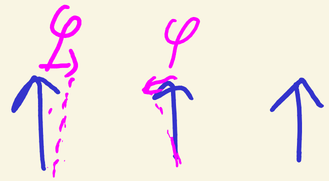
magnon is a boson field

particle = 

Holstein-Primakoff boson $S \to \infty$

Spin wave in uniform ferromagnet

S



$H_J = \frac{J}{2}\int dr\,(\nabla S)^2$ exchange energy favors uniform S

· spin wave (fluctuation)

$$S = \begin{pmatrix}\varphi_x \\ \varphi_y \\ S\end{pmatrix} + O(\varphi^2) \qquad |\varphi| << S \qquad \nabla S \sim \nabla \varphi, \quad \varphi = \begin{pmatrix}\varphi_x \\ \varphi_y\end{pmatrix}$$

$$\Rightarrow H_J \simeq \frac{J}{2}\int dr\,(\nabla\varphi)^2 = \sum_k \omega_k\, \varphi(k)^2$$

$\omega_k = \frac{J}{2}k^2$ energy of spin wave excitation (ferromagnetic)

(antiferro $\Rightarrow \omega_k \propto k$)

· dynamics

Landau-Lifshitz equation

$$\dot S = B \times S \qquad B = -\frac{\delta H}{\delta S} = \frac{J}{2}\nabla^2 S$$

$$\rightarrow \quad \dot S = -\frac{J}{2} S \times \nabla^2 S$$

$$\xrightarrow{SW} \quad \dot\varphi = -\frac{JS}{2}\hat z \times \nabla^2 \varphi$$

$$\dot\varphi_x(k) = \omega_k\,\varphi_y(k)$$
$$\dot\varphi_y(k) = -\omega_k\,\varphi_x(k)$$
$$\varphi_\pm = \varphi_x \pm i\varphi_y$$

$$\dot\varphi_\pm(k) = \mp i\,\omega_k\,\varphi_\pm(k)$$

Field theoretical representation

- Lagrangian

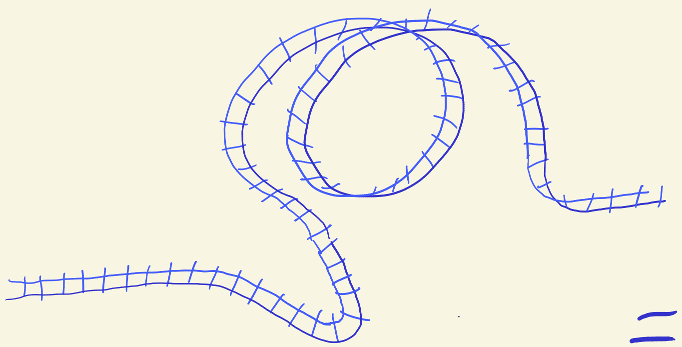
$$\mathcal{L} = \int dr \left[S(1-\cos\theta)\dot{\phi} - \frac{J}{2}(\nabla S)^2 \right]$$

- $$\frac{\vec{S}}{S} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} \simeq \begin{pmatrix} \varphi_x \\ \varphi_y \\ S \end{pmatrix} + O(\varphi^2)$$

$$\Rightarrow \mathcal{L} = \int dr \left[-\frac{i}{2} a^\dagger \overleftrightarrow{\partial_t} a - J|\nabla a|^2 \right]$$

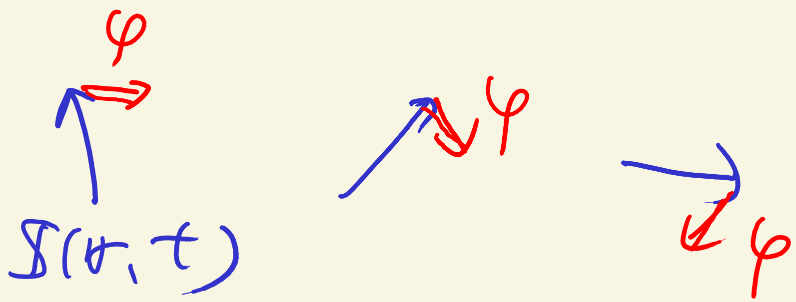
$$= \sum_k \left[-i a_k^\dagger \partial_t a - \omega_k a_k^\dagger a_k \right]$$

a boson with energy ω_k



for magnon

= magnetization structure



quantization axis for φ changes locally

⇒ unitary transformation

$$\mathbf{S}(\mathbf{r},t) = \underbrace{U(\mathbf{r},t)}_{3\times 3 \text{ rotation matrix}} \tilde{\mathbf{S}}$$

$$\tilde{\mathbf{S}} = \begin{pmatrix} \varphi_x \\ \varphi_y \\ S \end{pmatrix}$$ rotated frame

⇒ effective gauge field

$$A_\mu = -i U^{-1} \partial_\mu U$$

adiabatic component

$$A^z_\mu = (1-\cos\theta)\partial_\mu \phi$$

universal same as electron

effective gauge field

$$A_\mu = -i\, U^{-1} \partial_\mu U$$

gauge coupling

$$H_A = -\underbrace{j_m}_{\text{magnon current}} \cdot A$$

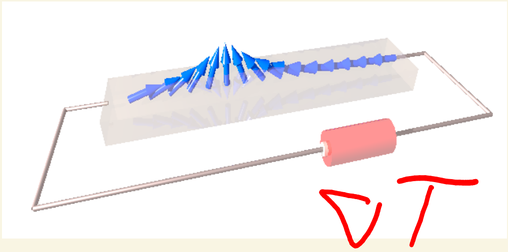
$\Rightarrow$ the same physics as conduction electron

- Spin transfer effect

  magnon current $j_m \Rightarrow$ magnetization flows

  - $j_m \propto \nabla T$ temperature gradient

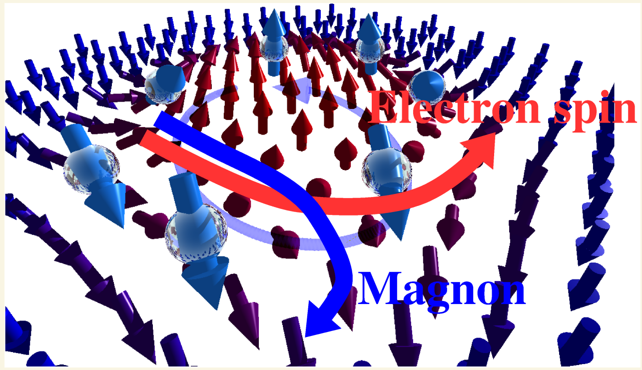
    no electric field to drive



∇T

  - magnon spin $= -1$ negative

- Hall effect due to effective magnetic field $B_s = \nabla \times A$



Effective gauge field for light

real gauge field in ferromagnet (localized spin) magnetization M

- Rashba gauge field $A_R = \alpha_R \times \sigma \Rightarrow \alpha_R \times M$

for electrons

strong sd

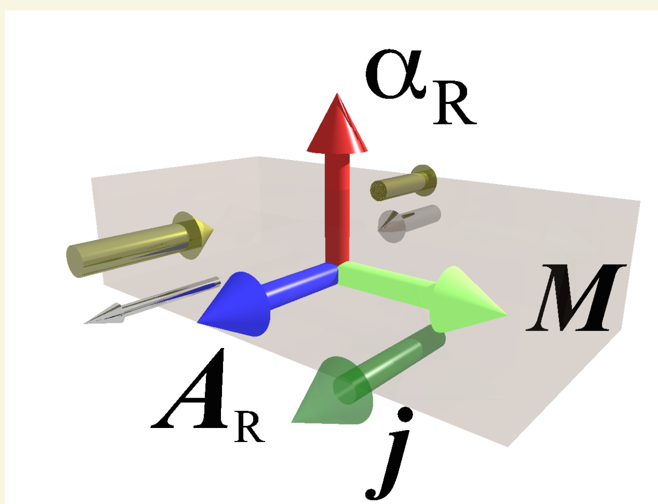
$\sigma \parallel M$

- Parity broken
- T-reversal broken

- A_R acts as a gauge field for light

A_R intrinsic electron flow

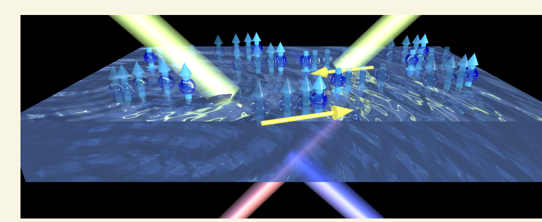
$\Rightarrow$ light gets Doppler shift



Directional dichroism

Asymmetric light propagation with respect to A_R

half mirror



Both photon and electron feel the same vector potential

$A_R = \alpha_R \times M$ troidal moment

Electromagnetism including Rashba effective gauge field

$A_R = \alpha_R \times M$ vector potential for light

$\Rightarrow$ $\mathbb{K}\cdot \mathbb{A}_R$ coupling for $\mathbb{K}$ of light $\mathbb{K} = \mathbb{E}\times\mathbb{B}$

Poynting vector

$\Rightarrow$ $H_{EM} = \mathbb{A}_R\cdot(\mathbb{E}\times\mathbb{B})$ Coupling between material and electromagnetic field

Kawaguchi GT 2016

$\Rightarrow$ $\mathbb{E}_{tot} = \mathbb{E} + \mathbb{A}_R \times \mathbb{B}$

$\mathbb{B}_{tot} = \mathbb{B} + \mathbb{A}_R \times \mathbb{E}$

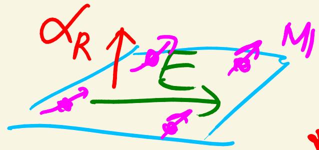
Lorentz transformation to a moving frame with velocity $\mathbb{A}_R$

consistent with $\mathbb{A}_R$ is an intrinsic flow

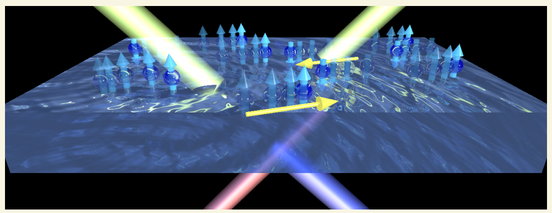
- Spin charge mixing EB mixing
- anomalous optical property dichroism

universally explained in terms of effective gauge field $\mathbb{A}_R$

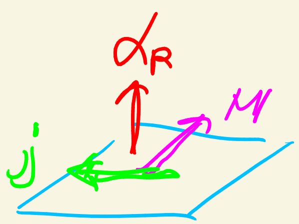
$M = \kappa_{ME}(\alpha_R \times \mathbb{E})$



A_R



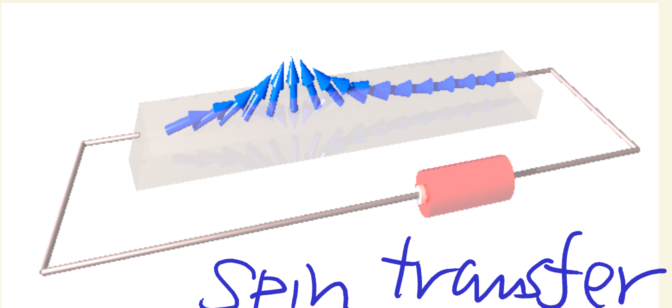
$j = \kappa_{ME}(\alpha_R \times \dot{B})$



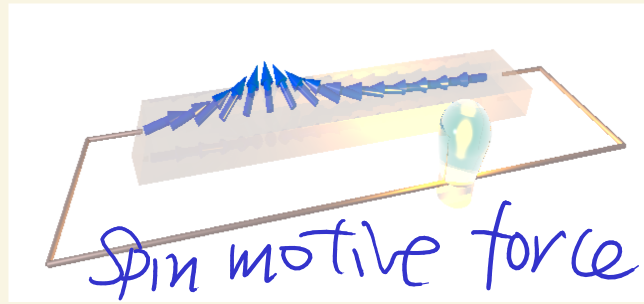
Effective gauge field* in metalic ferromagnet

* universal : electron, magnon, light, ...

• adiabatic



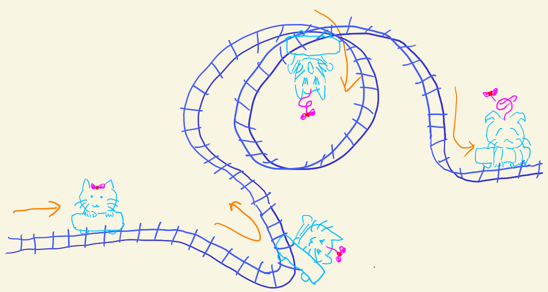
Spin transfer

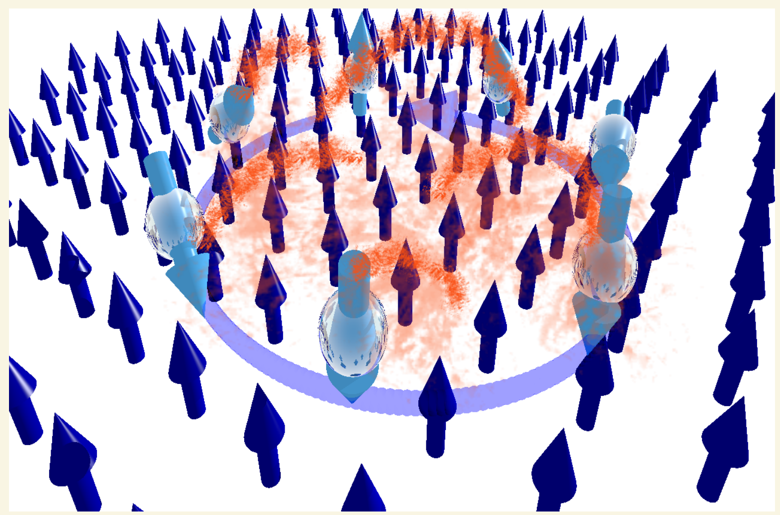


Spin motive force E_s

B_s

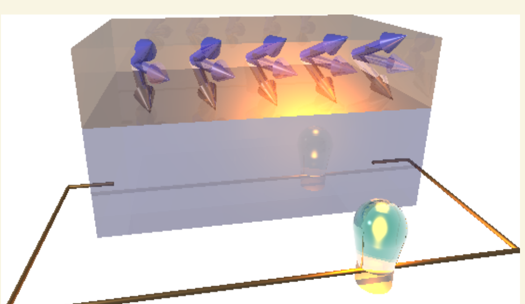
Spin Hall effect



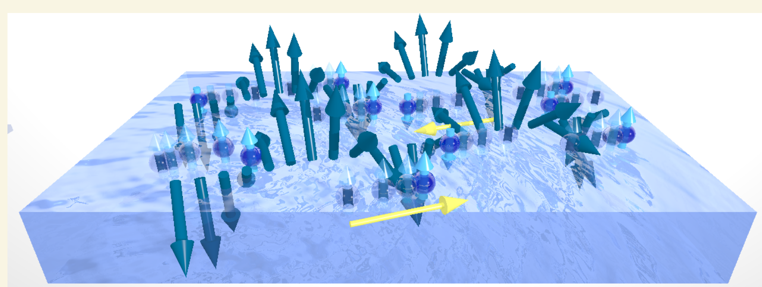


• nonadiabatic

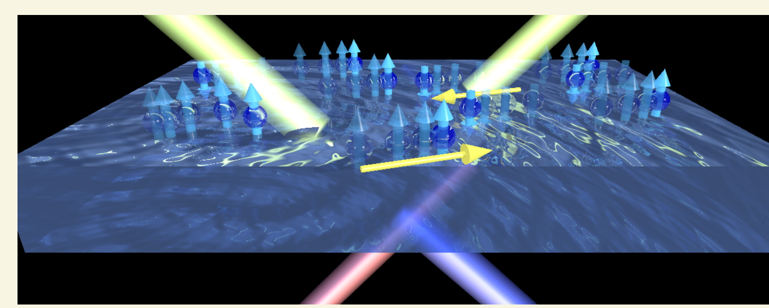
• with spin-orbit



Spin pumping



Dzyaloshinskii - Moriya spiral



Directional dichroism light